Chapter 6
Combinatorial Approach to Sensor Activation

As was elucidated in Chapter 3, although laborious research on the development of strategies for efficient sensor placement has been conducted with numerous contributions and the need for systematic methods was widely recognized, most techniques communicated by various authors usually rely on exhaustive search over a predefined set of candidate solutions and the combinatorial nature of the design problem is taken into account very occasionally [305]. Obviously, such an approach is feasible for a relatively small number of possible sensor locations, and becomes useless as the number of possible candidate locations increases.

The introduction of continuous designs presented in Chapter 3 leading to extremely effective algorithms solving the scheduling task via continuous relaxation, constitutes a possible approach with a great potential for applications. However, bearing in mind that it provides only approximate solutions to the problem, in some practical situations it may be difficult to control the level of suboptimality for the resulting activation schedules. Particularly, within the setting where numerous constraints related to network resources and costs of the experiment have to be imposed, a rounding of experimental design may significantly deteriorate the quality of the solution. Moreover, if the number of activated sensors does not necessarily have to be large, the exchange clusterization-free algorithms discussed in Sections 3.1.4 and 5.2 can fail to provide satisfactory approximation to the global optimum.

The aim of the research reported here was to develop a practical approach to sensor selection which would be versatile enough to cope with practical monitoring networks consisting of many discrete scanning sensors [183, 200, 204, 282, 291]. Specifically, we adopt the setting for which the observation system comprises multiple stationary sensors located at already specified locations, and it is desired to activate only a subset of them during a given time interval while the other sensors remain inactive [50]. A reason for not using all the available sensors could be the reduction of the observation system complexity, the cost of operation and maintenance and/or limited network resources [305]. Since selecting the best subset of sites to locate the...
sensors constitutes an inherently discrete large-scale resource allocation problem whose solution may be prohibitively time consuming, an efficient guided search algorithm based on the branch-and-bound method is developed, which implicitly enumerates all the feasible sensor configurations, using relaxed optimization problems that involve no integer constraints.

Obviously, this idea is not novel, since the branch-and-bound method constitutes one of the most frequent approaches to solve discrete optimization problems and it has indeed been used in the context of network design, cf., e.g., [25]. The key issue leading to effective application of the branch-and-bound technique is an estimation of accurate bounds for optimal solution at each node of the search tree. Uciński and Patan [291] developed a simple, yet powerful, computational scheme to obtain such bounds for the restricted problems in the case of stationary sensor networks. In their paper, apart of derivation of the optimality conditions, an original algorithm is proposed which can be interpreted as a simplicial decomposition one with the restricted master problem which can be readily solved by the weight optimization algorithms discussed in Section 3.1.2. The proposed combination of the simplicial decomposition algorithm with the known weight optimization algorithm for constructing optimizing probability distributions constitutes a novel approach which is very flexible for different extensions regarding scanning observations [183, 282], parallelization of computations [183], temporal or experimental constraints [204, 205] and a multiobjective formulation [206].

6.1 Scanning Problem Revisited

Here, the fixed switching schedule for scanning sensors introduced in Section 3.1 is adopted, i.e., the time horizon $T$ is arbitrarily partitioned into subintervals $T_k = (t_{k-1}, t_k]$, $k = 1, \ldots, K$, where $0 = t_0 < t_1 < \cdots < t_K = t_f$. The state $y$ can be observed (possibly indirectly) by $N$ pointwise sensors, but among them only $n_k$ are activated on $T_k$.

The optimal sensor scheduling problem considered in what follows consists in seeking for each time subinterval $T_k$ the best subset of $n_k$ locations from among the $N$ given potential ones, so that the problem is reduced to a combinatorial one. In other words, the problem is to divide for each time subinterval the $N$ available sensor nodes into $n_k$ active ones and the remaining $N - n_k$ dormant ones so as to maximize a criterion quantifying the identification accuracy. In order to formulate this mathematically, introduce for each possible location $x^i$ ($i = 1, \ldots, N$) a set of variables $v^i_k$, each of them taking the value 1 or 0 depending on whether or not a sensor residing at $x^i$ is activated during $T_k$, respectively. The FIM in (3.2) can then be rewritten as

$$M(v_1, \ldots, v_K) = \sum_{i=1}^{N} \sum_{k=1}^{K} v^i_k M^i_k,$$

(6.1)