Symbolic Coupling Traces for Coupling Analyses of Medical Time Series

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Abstract. Directional coupling analysis of bivariate time series is an important subject of current research. In this contribution, a method based on symbolic dynamics for the detection of time-delayed coupling is reviewed. The symbolic coupling traces, defined as the symmetric and diametric traces of the bivariate word distribution, are applied to model systems and cardiological data as well as sleep data showing its advantages especially for nonstationary data.

1 Introduction

The cardiovascular systems consist of several subsystems which are interrelated by feedback loops with time delay. Revealing such time delayed coupling directions is a basic task in understanding the system. Different methods, starting from cross correlation via mutual predictability to information-theoretic approaches are proposed for this purpose, but, due to the non-stationarity, non-linearity, and the noise, the conclusions are not homogeneous. In this paper, applications of an enhanced method based on symbolic dynamics [1] for the detection of time-delayed coupling is reviewed. The symbolic coupling traces allow for a more robust analysis of delayed coupling directions between heart rate and blood pressure [2, 3, 4].

2 Materials and methods

For bivariate coupling analysis we used the method of symbolic coupling traces (SCT) [2]. First step of this approach is the transformation of the time series of beat-to-beat intervals $B_i$ and systolic blood pressure $S_i$, into symbol sequences $s_B(t)$ and $s_S(t)$ according to the rule

\begin{equation}
\begin{cases}
1, z(t) \leq z(t + \theta) \\
0, z(t) > z(t + \theta)
\end{cases}
\end{equation}

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where \( z \) represents \( B \) and \( S \). For analysis of short-term couplings in \( B_i \) and \( S_i \) the value \( \theta = 1 \) has been used [2]. Next, words of length \( l \) are constructed \( w_z(t) = s(t), s(t+1), ..., s(t+l-1) \) which can form \( d = 2^l \) different patterns. For short-term dynamics in \( B_i \) and \( S_i \), \( l = 3 \) is used to reliably estimate the bivariate word distribution [2]. \( w_x(t) \) and \( w_y(t) \) are used to calculate the bivariate word distribution 
\[
\begin{align*}
\pi_{ij}(\tau) &= P(w_x(t) = W_i, w_y(t + \tau) = W_j) \\
&= P_{ij}(\tau)
\end{align*}
\]
with the \( d \) patterns \( W_1 \) to \( W_d \). The parameter \( \tau \) is included in order to consider delayed interrelationships between the signals. From the bivariate word distribution, the parameters
\[
\begin{align*}
T &= \sum_{i=j} \pi_{ij}(\tau) \\
\overline{T} &= \sum_{i=1, ..., d; j=d+1-i} \pi_{ij}(\tau) \\
\Delta T &= T - \overline{T}
\end{align*}
\]
are calculated. On the one hand, \( T \) only captures influences which preserve the structure of the transmitted pattern of dynamics (symmetrical influences). On the other hand, \( \overline{T} \) only quantifies influences which invert the dynamical structure of the driver (diametrical influence). To answer the question if the parameter \( \Delta T \) is significant or not, a critical value \( \Delta T_{\text{crit}} \) is estimated respectively. Therefore, these parameters are calculated for 1000 realizations of bivariate white noise \( N_i(0, \sigma^2) \) with sample length \( N \). We look for the 99th percentile where 99\% of the 1000 observation are smaller than that critical value. It represents the critical value of the significance level \( \alpha = 0.01 \) in an one-side significance test. The nonlinear regression leads to \( \Delta T_{\text{crit}}(N) \approx \pm 2.7 \cdot N^{-0.51} \).

3 Results

The SCT-method was applied to real cardiological data to analyze the coupling between \( B_i \) and \( S_i \) of 20 healthy volunteers (age: 53.0 ± 8.0 years). For all subjects, we measured continuous blood pressure signals (30 min, Portapres Mod. 2, 100 Hz sampling frequency, under standardized supine resting conditions, recorded at the Charité Berlin). A representative example of the coupling analysis is shown in Fig. 1. Parameters based on SCT are not influenced by instationarities of the time series. The parameter \( \Delta T \) detects significant lags at \( \tau = -2 \) and \( \tau = 0 \) for all subjects. This confirms the prevailing opinion about the cardiovascular short term regulation. The symmetric lag at \( \tau = 0 \) reflects the mechanically induced arterial pressure fluctuations, whereas the diametric lag at \( \tau = -2 \) represents the vagal feedback from the \( B_i \) to the \( S_i \).

Further on, we considered polysomnographical measurements of 18 normotensive (NT, age: 44.6 ± 7.6 years, BMI: 30.2 ± 2.9 kg/m\(^2\), all male) and 10