Chapter 6
Fundamentals of Robust Control

‘A sign of great art is often its openness to multiple readings depending on the observer’s perspective.’ — Tariq Samad, Former Editor-in-Chief of IEEE Control Systems Magazine

Summary:
This chapter is dedicated to theoretical basics of robust control. In this chapter new concepts are introduced, such as:
• internal stability of the control system
• controller parametrization such that internal stability holds
• robust stability of the control system
• robust performance.

In this chapter the fundamentals of the robust control systems are presented. Until now the controller has been designed for the nominal plant. However, using simulation has been verified and validated considering plant uncertainty, too.

One of the main differences between the classical control design and robust control design is that in case of later one, plant uncertainty is considered explicitly during design. We will try to illustrate the main concepts as well as how the robust control problem is formulated. However, for controller design (for solving the robust control problem) we will need advanced software tools.

In this chapter we will specify the performance of a control system (amplitude of certain signals of interest) using norms. Therefore, first we briefly recall the properties of norms and then we will introduce the $\| \cdot \|_\infty$ norm.

6.1 Review of Norms for Signals and Systems

The properties of a norm are [25]:
• $\| u \| \geq 0$
• $\| u \| = 0 \iff u(t) = 0, \forall t$
• $\| au \| = |a| \| u \|, \forall a \in \mathbb{R}$
• $\| u + v \| \leq \| u \| + \| v \|$ triangle inequality

### 6.1.1 Norms for Signals

The **1-norm** of a signal $u(t)$ is the integral of its absolute value:

$$\| u \|_1 := \int_{-\infty}^{\infty} |u(t)| dt \quad (6.1)$$

The **2-norm** of a signal $u(t)$ is:

$$\| u \|_2 := \left( \int_{-\infty}^{\infty} u^2(t) dt \right)^{1/2} \quad (6.2)$$

We remark that the instantaneous power of a signal $u(t)$ is defined as $u^2(t)$ and its energy is defined as the square of its 2-norm namely, $\| u \|_2^2$.

The **$\infty$-norm** of a signal is the least upper bound of its absolute value:

$$\| u \|_\infty := \sup_t |u(t)| \quad (6.3)$$

For example, the $\infty$-norm of the $u(t) = 1 - e^{-t}$ signal equals 1.

### 6.1.2 Norms for Systems

We consider systems that are linear, time-invariant, casual and finite-dimensional. The system’s transfer function is noted by $G(s)$. We introduce two norms for the transfer function $G(s)$.

The **2-norm** of $G(s)$ transfer function is:

$$\| G \|_2 := \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(j\omega)|^2 d\omega \right)^{1/2} \quad (6.4)$$

The **$\infty$-norm** of $G(s)$ transfer function is:

$$\| G \|_\infty := \sup \ |G(j\omega)| \quad (6.5)$$

**Theorem 6.1.** The 2-norm of $G(s)$ is finite iff $G(s)$ is strictly proper and has no poles on the imaginary axis; the $\infty$-norm is finite iff $G(s)$ is proper and has no poles on the imaginary axis.

**How to compute the $\infty$-norm?**

This requires a search. Set up a fine grid of frequency points $\omega_1, \ldots, \omega_N$. Then an estimate $\|G(s)\|_\infty$, which is $\max |G(j\omega)|$. Another way to find the maximum of $|G(j\omega)|$ is solving the equation:

$$\frac{d |G(s)|^2}{d\omega}(j\omega) = 0 \quad (6.6)$$