Chapter 2
Linguistic Aggregation Operators

When a problem is solved using linguistic information, it implies the need for computing with words (Zadeh and Kacprzyk, 1999a; 1999b). Thus, how to fuse the input linguistic information is an interesting and important research topic. Linguistic aggregation operators are a powerful tool to deal with linguistic information. Over the last decades, many scholars have focused their investigation on linguistic aggregation techniques and various linguistic aggregation operators have been proposed (Xu, 2007b), including the linguistic max and min operators (Xu and Da 2003b; Yager, 1998; 1996; 1995; 1992), linguistic median operator (Yager, 1998; 1996), Linguistic weighted median operator (Yager, 1998; 1996), linguistic max-min weighted averaging operator (Yager, 1995), extension principle-based linguistic aggregation operator (Bonissone and Decker, 1986; Bordogna and Passi, 1993; Chang and Chen, 1994; Chen, 1997; Degani and Bortolan, 1988; Delgado et al., 1998; Lee, 1996), symbol-based linguistic aggregation operator (Delgado et al., 1993a), 2-tuple arithmetic mean operator (Herrera and Martínez, 2000a), 2-tuple weighted averaging operator (Herrera and Martínez, 2000a), 2-tuple OWA (orded weighted averaging) operator (Herrera and Martínez, 2000b), linguistic weighted OWA operator (Torra, 1997), linguistic averaging operator (Xu, 2006b; 2005b; 2004d; 2004f), linguistic weighted disjunction operator (Herrera and Herrera-Viedma, 2000b; 1997), linguistic weighted conjunction operator (Herrera and Herrera-Viedma, 2000b; 1997), linguistic weighted averaging operator (Herrera and Herrera-Viedma, 1997; Xu, 2006b; 2005b; 2004d), ordinal OWA operator (Bordogna et al., 1997; Yager, 1995; 1992), ordinal linguistic aggregation operator (Delgado et al., 1993b), ordinal hybrid aggregation operator (Xu, 2004d), linguistic OWA operator (Delgado et al., 1998; Herrera and Herrera-Viedma, 2000a; Herrera et al., 2001b; 1997; 1996b; 1995; Xu, 2006a; 2006b; 2004b; 2004e; Xu and Da, 2003b), inverse-linguistic OWA operator (Herrera and Herrera-Viedma, 1997; Herrera et al., 2001b), linguistic hybrid aggregation operator (Xu, 2006a), induced linguistic OWA operator (Xu, 2006b), uncertain linguistic averaging operator (Xu, 2006b; 2004a), uncertain linguistic weighted averaging operator (Xu, 2006b), uncertain linguistic OWA operator (Xu, 2006b; 2004a), induced uncertain linguistic OWA operator (Xu, 2006b; 2004c), uncertain linguistic hybrid aggregation operator (Xu,
2004a), dynamic linguistic weighted averaging operator (Xu, 2007d), dynamic linguisti-
cric correlated averaging operator and linguistic correlated geometric operator (Xu, 2009b), etc. These linguistic aggregation operators have been studied and applied in a wide variety of areas, such as engineering (Levrat et al., 1997; Xu, 2004d), decision making (Bordogna et al., 1997; Chen and Hwang, 1992; Delgado et al., 2002; 1998; 1994; 1993a; 1993b; Herrera and Martínez, 2001a; Herrera et al., 2000; 1997; 1996a; 1996b; 1995; Herrera and Herrera-Viedma, 2003; 2000a; 2000b; 1997; Herrera and Martínez, 2001a; 2000b; Huynh and Nakamori, 2005; Xu, 2007a; 2006a; 2006c; 2006d; 2006e; 2006f; 2006g; 2006h; 2006i; 2005a; 2005b; 2005c; 2004a; 2004b; 2004c; 2004d; 2004e; 2004f; Tong and Bonissone, 1980; Yager, 1995; Yager and Kacprzyk, 1997), information retrieval (Bordogna and Passi, 1993; Delgado et al., 2002; Herrera-Viedma, 2001; Herrera-Viedma et al., 2003; Herrera-Viedma and Peis, 2003; Kostek, 2003), marketing (Herrera et al., 2002; Yager et al., 1994), scheduling (Adamopoulos and Pappis, 1996), biotechnology (Chang and Chen, 1994), materials selection (Chen, 1997), software system (Lee, 1996), personnel management (Herrera et al., 2001b), educational grading system (Law, 1996), medical diagnosis (Becker, 2001; Degani and Bortolan, 1988), artificial intelligence (Zadeh and Kacprzyk, 1999a; 1999b), supply chain management and maintenance service (Xu, 2004d), etc. Xu (2007b) provided a comprehensive survey of the existing main linguistic aggregation operators.

2.1 Linguistic Aggregation Operators Based on Linear Ordering

Let \((a_1, a_2, \cdots, a_n)\) be a collection of linguistic arguments, and \(a_j \in S_1 (j = 1, 2, \cdots, n)\). Yager (1998; 1996; 1995; 1992) introduced a linguistic max operator:

\[
LM_1(a_1, a_2, \cdots, a_n) = \max_j \{a_j\}
\]  
(2.1)

and a linguistic min (\(LM_2\)) operator:

\[
LM_2(a_1, a_2, \cdots, a_n) = \min_j \{a_j\}
\]  
(2.2)

Yager (1998; 1996) developed a linguistic median (\(LM_3\)) operator:

\[
LM_3(a_1, a_2, \cdots, a_n) = \begin{cases} 
  \frac{b_{n+1}}{2}, & n \text{ is odd} \\
  \frac{b_n}{2}, & n \text{ is even}
\end{cases}
\]  
(2.3)

where \(b_j\) is the \(j\)th biggest of \(a_i (i = 1, 2, \cdots, n)\).

These three operators are the simplest linguistic aggregation operators, which are usually used to develop some other operators for aggregating linguistic information.