3 Waveguide Discontinuities and Components

Abstract. The considered in this Chapter method of treatment of waveguide discontinuities is with the theory of diffraction of modes at the obstacles in waveguides and transmission lines, which gives clear understanding of this effect. The modes in waveguides are associated with the equivalent transmission lines, and the one-modal approximation is used widely in microwave techniques. The multimodal representation of the diffracted fields requires more complicated matching of them at the discontinuities, and the integral and stationary functional methods are used to obtain the equivalent circuit models of the obstacles. This idea, related to the founders of the waveguide theory, is additionally required for interpretations of the results obtained by those numerical methods which calculate the field in the whole discontinuity domain without using the modal expansion method. References -35. Figures -7. Pages -24.

3.1 A More Detailed Theory of Regular Waveguides for the Discontinuity Treatment

Very complicated effects appear when a waveguide mode encounters an obstacle or deformation of the waveguide shape. Similarly to the propagation of modes in uniform waveguides, the Maxwell or wave equations describe this effect and it can be solved by the above-considered methods modified for the 3-D or 4-D simulations.

Some of them provide clear understanding of the EM phenomena caused by diffraction of waves. An introduction to these methods requires more detailed explanation of the modal theory of waveguides and its relation to the formalism of the equivalent transmission lines and lumped circuits used to explain and model the waveguide discontinuities and components. Below, some of the most important details are explained based on the contributions of Collin [1], Felsen & Marcuvitz [2], Marcuvitz [3], Schwinger [4], Mashkovzev, et al. [5], Nikolskyi & Nikolskaya [6], Heaviside [7], Shelkunoff [8], et al.

Initially, a theory of discontinuities was established for the shielded transmission lines. The modal spectrum of these waveguides is discrete, i.e. it is infinite, but countable. At discontinuities, these modes are coupled to each other, and the diffraction field consists of all modes of the waveguide spectrum. Neglecting this effect or an asymptotic way of taking into account leads to the unimodal models of the discontinuities, and the method of equivalent lines and circuits is used in its simplest variant.
Consider again a shielded transmission line (Fig. 2.1). The field components $\mathbf{H}_i$ and $\mathbf{E}_i$, which are transversal to the propagation direction $0 \to z$, are expressed as

\[
j k_0 \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} E_z = \nabla \times (\mathbf{H}_i \times \mathbf{z}_0),
\]

\[
j k_0 \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} H_z = \nabla \times (\mathbf{z}_0 \times \mathbf{E}_i).
\]

In general, when the geometry of a line is varied with the longitudinal coordinate, the waveguide field consists of infinite sum of coupled modes. They can have all six field components, and it is more convenient to derive the expressions for the transversal fields $E_i$ and $H_i$ instead of the longitudinal ones $E_z$ and $H_z$:

\[
E_i = \sum_i V'_i(z) e'_i(x, y) + V''_i(z) e''_i(x, y),
\]

\[
H_i = \sum_i I'_i(z) h'_i(x, y) + I''_i(z) h''_i(x, y).
\]

The modal electric $(e'_i, e''_i)$ and magnetic $(h'_i, h''_i)$ functions are derived from

\[
e'_i = -\nabla \times \Phi_i,
\]

\[
h'_i = \mathbf{z}_0 \times e'_i,
\]

and

\[
e''_i = \mathbf{z}_0 \times \nabla \times \Psi_i,
\]

\[
h''_i = \mathbf{z}_0 \times e''_i.
\]

The scalar functions $\Phi$ and $\Psi$ are the solutions of the following boundary value problems:

\[
\nabla^2 \cdot \Phi_i + (\chi')^2 \Phi_i = 0,
\]

\[
\Phi_i(x, y) = 0, \quad (x, y) \in L
\]

and

\[
\nabla^2 \cdot \Psi_i + (\chi'')^2 \Psi_i = 0,
\]

\[
\frac{\partial \Psi_i(x, y)}{\partial n} = 0, \quad (x, y) \in L.
\]

The case $\chi^2 = k_0^2 \varepsilon \mu_r - k_z^2 = 0$ corresponds to the multiply connected cross-sections supporting the TEM or quasi-TEM modes. Then to the above-considered solutions of (3.5) or (3.6), the functions obtained from the Laplace equations $\nabla^2 \cdot \Phi = 0$ or $\nabla^2 \cdot \Psi = 0$ should be added to the modal field expansion (3.2).