Solventy Capital Estimation and Risk Measures

Antoni Ferri1,*, Montserrat Guillén1, and Lluís Bermúdez2

1 Departament d'Econometria, Estadística i Economia Espanyola, Riskcenter-IREA, University of Barcelona, Spain
tonoferrri@ub.edu
2 Departament de Matemática Financera i Actuarial, Riskcenter-IREA, University of Barcelona, Spain

Abstract. This paper examines why a financial entity’s solvency capital estimation might be underestimated if the total amount required is obtained directly from a risk measurement. Using Monte Carlo simulation we show that, in some instances, a common risk measure such as Value-at-Risk is not subadditive when certain dependence structures are considered. Higher risk evaluations are obtained for independence between random variables than those obtained in the case of comonotonicity. The paper stresses, therefore, the relationship between dependence structures and capital estimation.

Keywords: Solvency II, Solvency Capital Requirement, Value-at-Risk, Tail Value-at-Risk, Monte Carlo, Copulas.

1 Introduction

Recent years have seen the developement of regulatory frameworks designed to guarantee the financial stability of banking and insurance entities around the world. Europe developed Basel II and, more recently, Basel III for its banking market, and parallel to these accords drew up the Solvency II directive in 2010 for its insurance market and the Swiss Solvency Test in Switzerland. The regulatory frameworks seek to establish what might be considered a reasonably amount of capital (referred to as Solvency Capital in Basel II and III and as Solvency Capital Requirement in Solvency II and the Swiss Solvency Test) to put aside to ensure financial stability in the case of adverse fluctuations on losses. This quantity must reflect the entity’s specific risk profile and, under the aforementioned frameworks, it can be arrived at by applying either the Standard Model proposed by the regulator or an Internal Model proposed by the entity itself. In the later case, a number of requirements must first be satisfied before the model can be used for the purposes of capital estimation. In the European frameworks is regulated by the calibration of a risk measurement given a confidence level over a given time horizon. In this paper we focus our attention on the European insurance market, and more specifically in non-life underwriting risk, in relation to the Solvency II and Swiss Solvency Test regulations. Under both frameworks, the

* Corresponding Author.
Solvency Capital Estimation and Risk Measures

35

total capital estimation is obtained by aggregating individual capitals requirements arising from a company’s various sources of risk, based on the correlation between them as defined by the Standard Model. As a means of aggregating risks, we proposed a simulation of a multivariate random variable where each marginal distribution function represents the claims of a given line of business. We simulate a sample of this multivariate random variable taking into account the correlation between lines of business and, then, we aggregate the results of each simulated claim by line of business in order to obtain the distribution of the total claims. Finally, we estimate the capital requirements by applying a risk measure over the total claims distribution.

By representing an example of a multivariate random variable simulation we show that under certain assumptions there are risk measures that fail to satisfy the subadditive property. This is tipically the case where there is a very heavy tailed or skewed distribution on the margin and/or in which a special dependence structure is assumed for its joint distribution. Such circumstances can lead to an underestimation of the solvency capital if we incorrectly assimilate a risk measurement to the capital requirements, i.e., the appropriate distribution is not fitted to the marginals or the joint behavior of the marginal distributions is unknown.

The paper also emphasizes typical misunderstanding in the meaning of risk measures and the relationship between these risk measures and the underlying dependence structures of the variables, which represents the sources of risk.

2 Misunderstanding in the Concept of Value at Risk as a Risk Measure

Risk measures are tipically employed to determine the amount of capital that should be aside to cover unexpected losses. In [1] was proposed a number of desirable properties that a risk measure should satisfy in order to be considered a coherent risk measure. One such property, that of subadditivity, captures the idea of diversification across random variables since in the words of the authors: “...the merger [of two risks] does not create extra risk”. Suppose we have $n$ random variables, $X_i, i = 1, \ldots, n$ and its sum, $S = \sum_{i=1}^{n} X_i$, then we say that risk measure $\rho$ has the subadditivity property if and only if for all $X_i$

$$\rho(S) \leq \sum_{i=1}^{n} \rho(X_i).$$  \hspace{1cm} (1)

Although several risk measures are available, here we focus on two that are on loss distribution, namely Value-at-Risk and Tail Value-at-Risk. These risk measures seek to describe how risky a portfolio is. Of the two, the most frequently adopted is the Value-at-Risk measure given that it is employed under Basel III and Solvency II as a tool for calibrating the solvency capital requirements. Value-at-Risk and its properties has been widely discussed (see [2]). The Value-at-Risk measure is simply a quantile