Extractability as the Deduction Theorem in Subdirectional Combinatory Logic*

Hiroko Ozaki and Daisuke Bekki**

Ochanomizu University,
Faculty of Science, Department of Information Science,
2-1-1 Ohtsuka, Bunkyo-ku, Tokyo 112-8610, Japan

1 Introduction: Subdirectional Combinatory Logic and the Curry-Howard Isomorphism between Grammars, Term Calculi and Logics

The formulation of Combinatory Categorial Grammar (CCG) [7], especially the choice of combinatory rules and their nominatum, strongly imply connection with a typed-version of Combinatory Logic (CL). Since typed CL is a term calculus for an implication fragment of a Hilbert-style proof system, in the sense of the Curry-Howard isomorphism, it seems plausible to regard CCG as a grammar that corresponds to a Hilbert-style proof system, in that the associative Lambek calculus [3] corresponds to a Gentzen-style proof system.

This correspondence was not, however, as strictly established as expected. The main difference between CCG and CL is that the different linear orders of words are distinguished in CCG (as a grammar), that is, the exchange rule is not fully available in the contexts (as a proof system), which is reflected in the existence of two functional application rules. This is also true in Lambek lambda calculus, and gives rise to the emergence of two different lambda operators and function application constructions, in order to maintain the parallelism between the terms and their types. In CCG, however, this parallelism is not pursued so rigorously: CCG adopts simply-typed lambda calculus for its semantic representations and says nothing about the directionality of the lambda operators.

Subdirectional Combinatory Logic (SDCL) [1] is a term calculus that exactly corresponds to CCG. It is a kind of CL with directionality-sensitive combinators. More precisely speaking, SDCL is a class of logics containing various instances of CCG for each individual language, and we conjectured that the language variation can be described in terms of availability/absence of each directional

* We are grateful to Nicholas Asher, Frank Veltman, Alastair Butler, Shunsuke Yatabe and other audience of LENLS8 for their valuable comments and suggestions for the earlier version of this paper. We are also grateful to Kei Yura in Joint Research Symposium 2011.

** Daisuke Bekki is partially supported by Grant-in-Aid for Young Scientists (A), 22680013, 2010-203, from the Ministry of Education, Science, Sports and Culture, Japan.
combinator. The definition of SDCL that we use throughout this paper is given in the Appendix A.

The Curry-Howard isomorphism between a grammar and a term calculus or logic affords many advantages in the study of formal grammar. In Lambek calculus, for example, many results and methods of proof theory, such as cut-elimination, subformula properties and decidability, have been utilized and have brought about fruitful insights. As an illustration of such advantages, this paper discusses the following issues:

1. The relation between the status of extractability in CCG and the Deduction Theorem (DT) in Hilbert-style proof systems, with respect to the correspondence obtained through SDCL.
2. The relation between DT and the structural rules in CCG.
3. The relation between the type-raising rule and the complex NP constraints in CCG.

2 Extraction in CCG

In lexical grammars, the term extraction signifies wh-movements in generative grammar, that is, the dislocation of a wh-NP to a non-argument position. The following sentence is an instance of extraction of who from the object position of loves:

*a boy who John thinks that Mary loves*

The sentence is acceptable and is also derivable in CCG. However, it is well known that extraction from a complex NP is not allowed in English (the complex NP constraint [6]), as illustrated below:

* a girl who John met a boy who loved

This sentence is unacceptable and is not derivable in CCG. However, if we allow a type-raising rule for CCG, the sentence becomes derivable in the following way:

* a girl who John met a boy who loved*