Rank Reversal as a Source of Uncertainty and Manipulation in the PROMETHEE II Ranking: A First Investigation

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Abstract. PROMETHEE II is an aggregating procedure based on pairwise comparisons for ranking alternatives evaluated on multiple criteria. As other outranking methods, PROMETHEE II does not satisfy the assumption of independence to third alternatives. In other words, the ranks of two given alternatives may be influenced by the presence of a third one. This phenomenon, also called rank reversal, can be viewed as a source of uncertainty on the final ranking. Additionally, it raises the natural question of possible rank manipulations by adding "well-chosen" alternatives. This problem is studied in the context of a simplified version of the PROMETHEE II method also known as the Copland score. A linear program is proposed to test whether there is a way to rank a given alternative at the first position by adding artificial ones. Simulations are used to quantify the likelihood of this possibility and to test if it can be avoided.

1 Introduction

We consider the problem of ranking a finite set of alternatives evaluated on several conflicting criteria. Many aggregating methods are available in the literature to address this question. Among them, we may cite AHP, ELECTRE or PROMETHEE.

It is of common knowledge that these methods do not respect the assumption of independence to third alternatives. In other words, the relative ranks of two given alternatives may be influenced by the presence of a third one. This phenomena is also known as rank reversal (even if different definitions of this concept seems to co-exist). We refer the interested reader to [2], [1], [6], [8], [9], [10] and [12] for AHP, to [5] and [7] for PROMETHEE and to [13] for ELECTRE. Of course, this effect may be viewed as a source of uncertainty and manipulation in the final ranking.

Some authors consider the dependance to any kind of third alternatives, to an alternative that is dominated by all the others, to a copy of a given alternative or even the influence of a non-discriminating criterion.
A first natural question is to investigate if the rank of a given alternative is robust or not with respect to the other alternatives. A second question is to wonder if a given alternative could eventually become first by introducing artificial ones. Naturally, those alternatives should be well-chosen with respect to the initial set of alternatives (and not randomly). In this paper, the uncertainty of the final ranking is studied by means of the number of new alternatives needed to make first any given alternative. A low number of artificial alternatives meaning a higher uncertainty of the alternative rank. Of course, this question is strongly related to the possibility of manipulation in MCDA ranking methods which is of crucial importance.

In this paper, we restrict ourselves to a simplified version of the PROMETHEE II method (usual preference functions and equally weighted criteria). This method is described in Section 2. A linear program is then developed to test whether it is possible to bring an alternative ranked at the j-th place to the first place by adding a precise number of artificial alternatives. This linear program is presented in Section 3. Finally, we proceed by simulations to quantify the likelihood of manipulation. The design of the experiments and results are discussed in Section 4.

2 Concepts and Notations

Let us consider a set of alternatives \( A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\} \) and a family of criteria \( g_k \) with \( k \in K = \{1, 2, \ldots, k, \ldots, q\} \). Without loss of generality, we assume that all the criteria are positive and have to be maximized. Let \( g_k(a_i) \) be the evaluation of alternative \( a_i \) with respect to criterion \( g_k \). Each pair of alternatives \( a_i, a_j \in A \) are compared by computing the preference index \( \pi(a_i, a_j) \) in the following way:

\[
\pi(a_i, a_j) = \frac{1}{q} \sum_{k \in K} 1_{B(a_i, a_j)}(k),
\]

where \( B(a_i, a_j) = \{l \in K : g_l(a_i) > g_l(a_j)\} \) and \( 1_{B(a_i, a_j)} : K \to \{0, 1\} \) is an indicator function.

Based on this index, the PROMETHEE net flow gives a score for each alternative \( a_i \in A \):

\[
\phi_A(a_i) = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} (\pi(a_i, a_j) - \pi(a_j, a_i)).
\]

This flow is related to the Copland score [4] and can be viewed as an extremely simplified version of the PROMETHEE II method because the preference function used to compute the flow is the usual one and the weights are supposed to be equal. The interested reader may refer to [3] and [11] for more details about PROMETHEE methods.