The Algorithm Selection Problem on the Continuous Optimization Domain⋆

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Abstract. The problem of algorithm selection, that is identifying the most efficient algorithm for a given computational task, is non-trivial. Meta-learning techniques have been used successfully for this problem in particular domains, including pattern recognition and constraint satisfaction. However, there has been a paucity of studies focused specifically on algorithm selection for continuous optimization problems. This may be attributed to some extent to the difficulties associated with quantifying problem "hardness" in terms of the underlying cost function. In this paper, we provide a survey of the related literature in the continuous optimization domain. We discuss alternative approaches for landscape analysis, algorithm modeling and portfolio development. Finally, we propose a meta-learning framework for the algorithm selection problem in the continuous optimization domain.

1 Introduction

A continuous optimization problem is such that, given a function $f : \mathbb{R}^n \mapsto \mathbb{R}$, we want to find $\mathbf{x}^\star = \arg\min f(\mathbf{x})$. When solved in a computer, a *search algorithm* samples from the very large but finite *search set*, $\mathcal{X} \subset \mathbb{R}^n$. Each observation $\mathbf{x}_i \in \mathcal{X}$ has an associated output value $y_i \in \mathcal{Y}$ such that $y_i \approx f(\mathbf{x}_i)$, where $\mathcal{Y} \subset \mathbb{R}$ is the *objective set*. The algorithm aims to find one or more candidate solutions $\mathbf{x}_o \in \mathcal{X}, y_o \approx f(\mathbf{x}_o)$, such that $|y_o - y^\star| \ll \delta$, where $y^\star = f(\mathbf{x}^\star)$ and $\delta \to 0$. It is expected that the algorithm produces a solution of acceptable quality after a bounded number of function evaluations. The opposite is known as *premature convergence*.

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Premature convergence is related to the nature of the search algorithm, as each algorithm exploits differently the information obtained by sampling f. Therefore, unless some restrictions are in place, it is optimistic to expect that an algorithm would work well across a wide range of functions [78]. Due to the plethora of available algorithms, it is non-trivial to know which one is able to exploit the information more efficiently [29]. This is an instance of the well known algorithm selection problem. In this paper we propose a framework based on meta-learning for the algorithm selection problem. For this purpose, we review the literature about the different stages of the new framework — namely landscape analysis, meta-learning models and algorithm portfolios. Then, we outline the requirements for implementation of the new framework.

The paper is organized as follow: Section 2 presents the algorithm selection problem for continuous optimization, and the related parameter tuning problem. Section 3 describes the characteristics that make an optimization problem difficult and it reviews different methods for landscape analysis. Section 4 discusses how machine learning techniques have been employed to solve the algorithm and parameter selection problems. Section 5 analyzes the related works in algorithm portfolio design. Section 6 presents our meta-learning based framework for the algorithm selection problem. Finally, Section 7 discusses avenues for further research.

2 Algorithm Selection

Rice [56] defined the algorithm selection framework as a loose methodology that relates problems and solution methods through performance and problem characteristics. This framework did not provide specific methods for implementation, which is one of the reasons it has not been thoroughly explored. However, in the last decades, meta-learning has been favored as implementation method with demonstrated success in different problem domains [62]. Meta-learning exploits data obtained from previous experiments by constructing models that can be used for prediction, using machine learning techniques [28]. Figure 1 presents a summary of this implementation adapted to continuous optimization problems. In this figure, $\mathcal{F}$ is the very large, amorphous, high dimensional and hard to define *function set*, for which $f \in \mathcal{F}$. Let $\mathcal{A}$ be the large and diverse *algorithm set*, and $a \in \mathcal{A}$ be one of the many algorithms capable of searching for $\mathbf{x}_o$ in $\mathcal{X}$. The cost of running a in f can be measured by a function $\rho(f,a)$. Let $\mathcal{P} \subset \mathbb{R}$ be the set of feasible values of $\rho(f,a)$, called the *performance set*. Then, the algorithm selection problem is to find $a_o = \arg\min \rho(f,a)$ with f constant. It is noteworthy to point out that this problem cannot be solved directly. Hence, let $\mathcal{C} \subset \mathbb{R}^m$ be the *set of function characteristics*. This set includes known attributes of f such as the dimension, but also measurements about the occurrence of certain structures known to pose difficulties for a [57, 73]. Characteristics are important as they provide some order and coherence to the complicated problem space by imposing a lower dimensional coordinate system [57]. Characteristics can be calculated through user defined functions known as *landscape analysis* methods, $c(\mathbf{x}, y)$. These functions should be designed such that varying complexities are