FEM Implementation of Micropolar Hypoplastic Model

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Summary. A constitutive model based on hypoplasticity and micropolar continuum is developed by complex tensor formulations. The new model is simpler than any existing micropolar models. A characteristic length is the only additional material parameter. This model is implemented into ABAQUS and solved by finite element method (FEM). The simulation result of a simple shear test is compared with results of discrete element method (DEM). The comparison shows that the rotations of particles can be fairly well predicted with the micropolar hypoplastic model.

Keywords: hypoplasticity, micropolar, constitutive model, granular materials.

1 Introduction

In spite of their discrete nature, granular materials can be reasonably well described by continuum models. Recently, hypoplastic model based on nonlinear tensorial functions has attracted much attention [2]. The classical hypoplastic model does not have any internal length scale and therefore cannot account for problems with scale dependence. In granular materials, however, if the domain size is of the order of the mean grain size, the underlying boundary value problem shows scale dependence to some extent. A good example is the formation of shear band in granular materials [1]. There are several approaches to endow the constitutive equation with a characteristic length, e.g. micropolar theory [3], strain gradient theory [8] and nonlocal continuum [9]. The micropolar theory considers the relationship between couple stress and curvature in addition to the stress strain relationship. However, due to the lack of experimental data of micro scale, the couple-curvature relationship cannot be obtained directly. In this paper, the classical hypoplastic model is enhanced with the micropolar variables with the help of a complex formulation [4].

2 Formulations

Hypoplastic model is a constitutive model in rate type, which can be expressed as nonlinear tensorial functions:

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\[ \mathbf{T} = L(\mathbf{T}) : \mathbf{D} + N(\mathbf{T}) \parallel \mathbf{D} \parallel \]  

where \( \mathbf{T} \) is the stress tensor, \( \mathbf{T} \) is objective rate and \( \mathbf{D} \) is the stretching tensor. \( L \) and \( N \) are tensorial functions of \( \mathbf{T} \) and can be obtained with the help of the representation theorems for isotropic tensorial functions. The development and calibration of the model are mainly based on the results of element tests with homogeneous stress and strain. Analog to complex numbers, tensors can also be put into a complex form \( \mathbf{A} + i\mathbf{B} \), where \( \mathbf{A} \) and \( \mathbf{B} \) are tensors and \( i \) is called the imaginary unit, where \( i^2 = -1 \). In this expression, \( \mathbf{A} \) is called the real part and \( \mathbf{B} \) is called the imaginary part of the complex tensor. If both \( \mathbf{A} \) and \( \mathbf{B} \) are nth order tensors, \( \mathbf{A} + i\mathbf{B} \) is a nth order complex tensor. In this way the complex tensors contain the ordinary real tensors while extending them in order to solve problems that cannot be solved with only real tensors. Hence, complex tensors are widely used in theoretical physics and continuum mechanics.

In a micropolar continuum, the statical and kinematical variables \( \mathbf{T}, \mathbf{T}, \text{ and } \mathbf{D} \) are augmented by the moment stress \( \mathbf{M} \), its objective rate \( \mathbf{M} \) and the curvature rate \( \mathbf{K} \). General stress and strain tensors in complex form are substitute into the hypoplastic model, the following complex tensors for the static and kinematic variables can be written out according to the above definition of complex tensors:

\[ \mathbf{T}' = \mathbf{T} + i\mathbf{M}/l, \quad \mathbf{T}' = \mathbf{T} + i\mathbf{M}/l, \quad \mathbf{D}' = \mathbf{D} + i\mathbf{K}l \]  

Dimension analysis requires that a characteristic length \( l \) need be introduced. The operations for real tensors, such as inner product are defined by Xiao ([5]). Inserting the complex tensors in (2) into the constitutive equation (1) gives rise to

\[ \mathbf{T}' = L(\mathbf{T}'): \mathbf{D}' + N(\mathbf{T}'): \mathbf{D}' \parallel \]  

All tensors in the above equation have two parts, a real and an imaginary part. By separating the real and imaginary part, two constitutive equations are obtained, one for the stress-strain variables and the other for the moment-curvature variables.

\[ \mathbf{T} = L_1(\mathbf{T}, \mathbf{M}, l) : \mathbf{D} + L_2(\mathbf{T}, \mathbf{M}, l) : \mathbf{K} + N_1(\mathbf{T}, \mathbf{M}, l) \sqrt{\| \mathbf{D} \|^2 + l^2 \| \mathbf{K} \|^2} \]  

\[ \mathbf{M} = L_3(\mathbf{T}, \mathbf{M}, l) : \mathbf{K} + L_4(\mathbf{T}, \mathbf{M}, l) : \mathbf{D} + N_2(\mathbf{T}, \mathbf{M}, l) \sqrt{\| \mathbf{D} \|^2 + l^2 \| \mathbf{K} \|^2} \]  

The ensuing equations (4) (5) are largely simplified by assuming that the tensors of stress, strain and their rates are symmetric, the tensors of moment stress, curvature and their rates are antisymmetric. These two simplified equations define the micropolar hypoplastic model. This model is simpler than the existing micropolar