Parsimony Pressure versus Multi-objective Optimization for Variable Length Representations

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Abstract. We contribute to the theoretical understanding of variable length evolutionary algorithms. Such algorithms are very flexible but can encounter the bloat problem which means solutions grow during the optimization run without providing additional benefit. We explore two common mechanisms for dealing with this problem from a theoretical point of view and point out the differences of a parsimony and a multi-objective approach in a rigorous way. As an example to point out the differences, we consider different measures of sortedness for the classical sorting problem which has already been studied in the computational complexity analysis of evolutionary algorithms with fixed length representations.

1 Introduction

Evolutionary algorithms that work with a variable length representation often encounter the bloat problem which means that individuals grow without providing additional benefit to the quality of the solutions. Even worse such a growth of the individuals can block the optimization process such that problems that are relatively easy to optimize can not be handled by variable length evolutionary algorithms. Due to this problem, different methods have been introduced to deal with the bloat problem. Our goal is to study the behavior of variable length evolutionary algorithms from a mathematical perspective. We will examine algorithms for distinguished classes of problems and point out the impact of different approaches for dealing with the bloat problem in a rigorous way.

The most prominent example of a variable length evolutionary algorithm is genetic programming [7] which often evolves tree structures for a given problem. Recently, the first computational complexity results on these type of algorithm have been obtained. They follow the line of research that has successfully followed for evolutionary algorithms with fixed length representation (see the books [1, 11] for an overview). Variable length representations increase the search space significantly and in the light of genetic programming it seems to be wishful to better understand the behavior of algorithms using such representations from a theoretical point of view.

The computational complexity analysis of variable length evolutionary algorithms has started just recently. For example, Cathabard et al. [2] investigated
non-uniform mutation rates for problems with unknown solution lengths. They used a simple evolutionary algorithm to find a bitstring with an unknown number of leading ones, and although the bitstring had some predetermined maximum length, only an unknown number of initial bits was used by the fitness function. Durrett et al. \[3\] investigated worst-case and average-case runtimes of a simple tree-based genetic programming algorithm. The tackled problems were separable, with independent and additive fitness structures. Kötzing et al. \[6\] analysed simple GP algorithms for the MAX problem.

One prominent way of dealing with the bloat problem is the parsimony approach. In the case, that two solutions have equal quality the solution of lower complexity is preferred. Another way of coping with the bloat problem is to use a multi-objective approach which uses in each iteration of a variable length evolutionary algorithm a population which represents the different trade-offs according to the original goal function and the complexity of a solution. The solutions that represent the trade-offs are called Pareto optimal. Note that the parsimony approach is a scalarization approach as it uses these Pareto optimality and a lexicographic ordering. It is known that each global solution is also Pareto optimal, but not all Pareto optimal solutions can necessarily be found through scalarizations (e.g., see \[13\]). Both approaches of coping with the bloat problem have recently been examined for the problems ORDER and MAJORITY in the context of genetic programming \[9, 14\].

We further explore the use of parsimony pressure and multi-objective models. In \[9\] it is shown that both approaches help for ORDER and MAJORITY, but the differences between these two approaches are not examined. In this paper, we point out that switching from the parsimony approach to the multi-objective one can significantly reduce the runtime. In particular, we show that the parsimony approach can have local optima which lead to an infinite runtime whereas the multi-objective approach is able to compute the optimal solution within a polynomial number of steps.

We show these results for a classical problem from the computational complexity analysis of evolutionary algorithms with fixed-length representations, namely the sorting problem (sorting). Scharnow, Tinnefeld, and Wegener \[12\] considered sorting as an optimization problem and investigated different fitness functions measuring the sortedness of a permutation of elements. Different fitness functions lead to problems of different difficulties. Our goal is to explore how variable-length evolutionary algorithms behave on these problems. We take it as a prominent example to discuss the differences between a parsimony approach and a multi-objective one. In particular, we show that the parsimony approach can end up for a lot of the different sortedness measures in local optima when using a variable length representation whereas the multi-objective approach allows to compute the whole Pareto front in expected polynomial time.

Our paper is organized as follows. In Section 2 we introduce the two models and the different measures of sortedness. We examine the parsimony approach in Section 3 and show that it leads to local optima in the search space. In Section 4 we show that the multi-objective approach is able to compute the whole