From mathematical point of view, fractional derivative $a f^{(\nu)}(x)$ of order $\nu$ is a function of three variables: the lower limit $a$, the argument $x$ and the order $\nu$. Naming this functional the derivative, we believe that in case of integer $\nu$, $\nu = n$, it coincides with the $n$-order derivative. Extending the interrelation $df^{(n)}(x)/dx = f^{(n+1)}(x)$ to negative values of order, we interpret $f^{(-m)}(x)$, $m > 0$, as integrals, and $f^{(0)}(x) = f(x)$. Now, the function $a f^{(\nu)}(x)$ can be considered as an analytic continuation of $f^{(n)}(x)$, $n = \ldots, -2, -1, 0, 1, 2, \ldots$ saving basic properties of multiple derivatives.

4.1 Riemann-Liouville fractional derivatives

Reading the first part, we had possibility to make sure in ubiquitous of heredity, nonlocality and selfsimilarity in nature. Combining these properties of observed phenomena, we choose the power function $\Phi_{\mu}$, $0 < \mu < 1$ as an influence function in a hereditary integral:

$$g(x) = \int_0^x \Phi_{\mu}(x - \xi) f(\xi) d\xi = \Phi_{\mu} \ast f(x).$$

Recall, that

$$\hat{g}(\lambda) = \Phi_{m}(\lambda) \hat{f}(\lambda) = \lambda^{-m} \hat{f}(\lambda), \quad m = 1, 2, 3, \ldots,$$

and consequently $\lambda^{-m} \hat{f}(\lambda)$ is the Laplace image of $m$-fold integral of function $f(x)$,

$$\lambda^{-m} \hat{f}(\lambda) = \mathcal{L} \{0 \text{l}^m_x f(x)\} (\lambda), \quad 0 \text{l}^m_x f(x) \equiv \int_0^x d\xi_m \int_0^{\xi_m} d\xi_{m-1} \cdots \int_0^{\xi_2} d\xi_1 f(\xi_1).$$

Thus, the product $\lambda^{-\mu} \hat{f}(\lambda)$ can be considered as an interpolation of $m$-fold integral to fractional multiplicity $\mu$:
The function $f$ we can write the following consequence of equalities:

$$
\lambda^{-\mu} \hat{f}(\lambda) = \mathcal{L}\{0_1^\mu f(x)\}(\lambda),
$$

$$
o_1^\mu f(x) = \frac{1}{\Gamma(\mu)} \int_0^x (x-\xi)^{\mu-1} f(\xi) d\xi = \Phi_\mu * f(x), \quad \mu > 0.
$$

Observe that change $\mu \mapsto \mu - 1$ in the integral is equivalent to action of the differentiation operator $D_x \equiv d/dx$. When $\mu \to 0$, the fractional integral $f_\mu(x)$ becomes the function $f(x)$ itself. Naively, one could expect that continuous passing $\mu$ from positive values to negative turns it into the derivative of a fractional order $\nu = -\mu$. However, the integral

$$
o_1^\mu f(x) = \frac{1}{\Gamma(\mu)} \int_0^x (x-\xi)^{\mu-1} f(\xi) d\xi
$$

with $\mu < 0$ diverges. Thus, we have to choose another way: leaving $\mu = 1 - \nu$ inside the interval $(0,1)$, to act on the integral by the operator $D_x$:

$$
o D_x^\nu f(x) = D_x o_1^{1-\nu} f(x) = \frac{1}{\Gamma(1-\nu)} D_x \int_0^x (x-\xi)^{-\nu} f(\xi) d\xi = D_x \Phi_{1-\nu} * f(x), \quad 0 \leq \nu < 1.
$$

This passage will look more natural, if we replace $o_1^\mu$, $\mu > 0$ by the symbol $o D_x^{-\mu}$:

$$
o D_x^\nu = D_x o D_x^{-1_{\nu}}, \quad 0 < \nu < 1.
$$

We shall continue to interpret the fractional integral of order $\mu$ as a fractional derivative of the negative order $\nu = -\mu$:

$$
o D_x^\nu f(x) = \frac{1}{\Gamma(-\nu)} \int_0^x (x-\xi)^{-\nu-1} f(\xi) d\xi = \Phi_{-\nu} * f(x), \quad \nu < 0.
$$

Using the notation

$$
o f^{(\nu)}(x) = o D_x^\nu f(x),
$$

we can write the following consequence of equalities:

$$
o f^{(\nu)}(x) = \Phi_{-\nu} * f(x), \quad \nu < 0,
$$

$$
o f^{(\nu)}(x) = D_x o f^{(\nu-1)}(x), \quad 0 \leq \nu < 1,
$$

$$
o f^{(\nu)}(x) = D_x^2 o f^{(\nu-2)}(x), \quad 1 \leq \nu < 2,
$$

$$
o f^{(\nu)}(x) = D_x^3 o f^{(\nu-3)}(x), \quad 2 \leq \nu < 3,
$$

$$
\ldots \ldots
$$

$$
o f^{(\nu)}(x) = D_x^n o f^{(\nu-n)}(x), \quad n - 1 \leq \nu < n,
$$

and so on.

For the sake of evidence, we consider this procedure in detail. Suppose, we have to compute the derivative $o f^{(3.4)}(x)$ (asterisk on $\nu$-axis, Fig. 4.1). We are able to find derivatives of any (integer or fractional) negative order and integer positive orders.