Chapter 18
Stability Analysis of Vector Equalization Based on Recurrent Neural Networks

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Abstract. Since the pioneer work of Hopfield on the computational capabilities of recurrent neural networks (RNNs), they have been applied to solve classification and optimization problems in many scientific disciplines. This can be done, either by using conventional training algorithms like back propagation through time, or by investigating the Lyapunov stability of these RNNs and comparing the corresponding Lyapunov function with the cost function of the optimization problem to be solved. The later method is especially interesting in the field of engineering because no training phase is needed, which is always associated with computational effort and time. In this chapter we focus on an application of RNNs in communications engineering, namely the vector equalization. The importance of this procedure arises from the fact that there is no need for training. The parameters of the RNN to act as vector equalizer can be obtained by investigating the stability properties of these networks and by choosing a suitable activation function, which will be the core of this work.

Keywords: Recurrent neural networks, stability analysis, vector equalization.

18.1 Organization of the Chapter

In Section 2 we introduce the vector-valued transmission model and present the problem of vector equalization. In Section 3 we discuss the recurrent neural networks (RNNs) with the corresponding state-space equations and revisit the Lyapunov theory on stability. The stability analysis of RNNs with time-invariant activation functions is considered in Section 4. Section 5 is dedicated to analyze the optimum activation function for the vector equalizer based on RNNs. In Section 6 we present the stability analysis of the RNN for time-variant activation functions in detail. The comparison between local and global stable vector equalizer

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based on RNNs is discussed in Section 7. We finish the chapter with a conclusion in Section 8.

The following notation is needed throughout the chapter. Vectors are underlined once, matrices twice, $(\cdot)^T$, $(\cdot)^H$ and $|\cdot|$ denote the transpose, conjugate transpose and the absolute value of a matrix or a vector. A matrix $B \geq 0$ ($B > 0$) means it is positive semidefinite (positive definite).

We restrict ourself to real-valued RNNs without hidden neurons. We will point out, where a result is also valid for complex-valued RNNs. Extension to complex-valued RNNs is in progress. Parts of this chapter have been published in ([17], [18], [19]).

### 18.2 Vector-Valued Transmission Model

The block vector-valued transmission model for linear modulation schemes without channel coding is shown in Fig. 18.1 and is described as follows [13], [14]:

$$\tilde{x} = r \cdot x + n_e \tag{18.1}$$

- $x$ is the transmit vector of size $(n \times 1)$
- $\tilde{x}$ is the receive vector of size $(n \times 1)$
- $\hat{x}$ is the soft-valued decided vector of size $(n \times 1)$ at the output of the vector equalizer, cf. Fig. 18.1
- $\hat{x}$ is the decided vector of size $(n \times 1)$, cf. Fig. 18.1
- $n_e$ is the colored noise vector of size $(n \times 1)$ with correlation matrix

$$\phi = \frac{N_0}{2} \cdot r \tag{18.2}$$

$N_0$ is the single-sided noise power spectral density

- $r$ is the discrete-time channel matrix of size $(n \times n)$. It is a symmetric $r = r^T$ and positive semidefinite matrix $r \geq 0$ because it is a correlation matrix. This property arises from the use of the channel matched filter at the receiver. $r$ depends on the transmission scheme (basic wave forms) and the channel. For coherent transmission (the case we are considering) a perfect knowledge of the channel impulse response and thus the discrete-time channel matrix $r$ at the receiver side is required.
- the channel matrix $r$ contains all physical properties of the transmission model.
- $\forall i \in \{1, 2, \cdots, n\}, \forall j \in \{1, 2, \cdots, M\}, M = 2^s, s \in \mathbb{N} / \{0\}$: $x_i \in A_x = \{a_1, a_2, \cdots, a_M\}, a_j \in \mathbb{R}$. In this case there are $M^n$ possible transmit vectors.

The vector-valued transmission model depicted in Fig. 18.1 is a general model and fits to different transmission schemes like OFDM (orthogonal frequency division

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1 For complex-valued case it is a hermitian matrix $r = r^H$.
2 For complex-valued case $a_j \in \mathbb{C}$. 