Chapter 4
3D Point Feature Representations

“One new feature or fresh take can change everything.”

NEIL YOUNG

In their native representation, points as defined in the concept of 3D mapping systems are simply represented using their Cartesian coordinates $x, y, z$, with respect to a given origin. Assuming that the origin of the coordinate system does not change over time, there could be two points $p_1$ and $p_2$, acquired at $t_1$ and $t_2$, having the same coordinates. Comparing these points however is an ill-posed problem, because even though they are equal with respect to some distance measure (e.g. Euclidean metric), they could be sampled on completely different surfaces, and thus represent totally different information when taken together with the other surrounding points in their vicinity. That is because there are no guarantees that the world has not changed between $t_1$ and $t_2$. Some acquisition devices might provide extra information for a sampled point, such as an intensity or surface remission value, or even a color, however that does not solve the problem completely and the comparison remains ambiguous.

Applications which need to compare points for various reasons require better characteristics and metrics to be able to distinguish between geometric surfaces. The concept of a 3D point as a singular entity with Cartesian coordinates therefore disappears, and a new concept, that of local descriptor, takes its place. The literature is abundant of different naming schemes describing the same conceptualization, such as shape descriptors, or geometric features, but for the remaining of this book they will be referred to as point feature representations.

The theoretical formulation of a feature representation for a given point $p_q$ is given as follows. Let $p_q$ be the query point, and $\mathcal{P}^k = \{p^k_1 \cdots p^k_2\}$ a set of points located in the neighboring vicinity of $p_q$. The concept of a neighbor is given as:

$$\|p^k_i - p_q\|_x \leq d_m$$

(4.1)

where $d_m$ is a specified maximum allowed distance from the neighbor to the query point, and $\| \cdot \|_x$ is an example $L_x$ Minkowski norm (though different other distance norms can be used). Additionally, the number of neighbors in $\mathcal{P}^k$ can be capped.
Fig. 4.1  Point feature representations for three pairs of corresponding points \((p_1, q_1)\) on 2 different point cloud datasets

to a given value \(k\). A point feature representation can then be described as a vector function \(F\), describing the local geometric information captured by \(\mathcal{P}^k\), around \(p_q\):

\[
F(p_q, \mathcal{P}^k) = \{x_1, x_2, x_3 \cdots x_n\}
\]  

(4.2)

where \(x_i, i \in \{1 \cdots n\}\) represents the dimension \(i\) of the resultant feature vector representation. Comparing two different points \(p_1\) and \(p_2\) then results in comparing the difference in some space between their feature vectors \(F_1\) and \(F_2\). Let \(\Gamma\) be the similarity measure describing the difference between \(p_1\) and \(p_2\), and \(d\) a distance metric, then:

\[
\Gamma = d(F_1, F_2)
\]

(4.3)

As \(d\) tends to some minimum, say \(d \to 0\), the two points will be considered as being similar with respect to their feature representations. If \(d\) is large, then the points are to be considered distinct from each other – i.e., they represent different surface geometries.

By including the surrounding neighbors, the underlying sampled surface geometry can be inferred and captured in the feature formulation, which contributes to solving the ambiguity comparison problem. Ideally, the resultant features would be very similar (with respect to some metric) for points residing on the same or similar surfaces, and different for points found on different surfaces, as shown in Figure 4.1.

A “good” point feature representation distinguishes itself from a “bad” one, by being able to capture the same local surface characteristics in the presence of: