The Membership Function and Its Measurement

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7.1 Introduction

Perhaps the most fundamental concept in fuzzy set theory is the membership function [24, 25]. Fuzzy sets allow for gradual degrees of membership and the membership function is a measure of that degree. The meaning of the membership function has been a question for everyone including Zadeh himself who advocated a linguistic approach [24-28] from the very early days. This idea also had support from linguists [3, 9] and psychologists [6].

In this chapter, we will take a closer look at the membership function as it relates to word meaning representation since the original idea of fuzzy sets has a strong affinity to linguistic scales and then discuss those studies that use the representational theory of measurement to provide meaning to the membership function. We argue that recent approaches to word meaning representation, a more complete theory of measurement which accounts for errors and empirical evidence that utilize brain imaging techniques can provide a much better understanding of fuzziness and a more coherent foundation to the theory. In passing, we identify potential areas of future research in this domain.

7.2 Meaning Representation in Linguistics

Since the semantics of fuzzy set theory is closely tied to the concept of a membership function one has to take a closer look at the meaning of the membership function. One possible direction is to discuss the meaning of a membership function in the context of word meaning representation. There are various approaches to word meaning representation [1]. The classical view of meaning representation stems from antiquity [18] and has been treated in formal logic since Aristotle until Wittgenstein forcefully argued that formal logic is not adequate to represent (word) meaning [22]. This view

\[ \text{It is widely accepted that word meaning (i.e., lexical-semantics) is grounded in conceptual knowledge. But one also needs to answer the more difficult question of whether the two are distinguishable from each other. There is empirical evidence on closeness of semantic and conceptual representations by brain imaging research. We do not delve into this debate here but note that most theories about the representation of meaning propose conceptual representations rather than semantic representations which implies that they take concepts and semantics to be the same.} \]

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was also shared by psychologists who argued that category boundaries are fuzzy [17]. Mainly there appeared two major alternative representations [21]: the holistic theories based on relations among words which evaluates meanings of words in relation to other words by using differential scales or network models, and featural theories where the representation relies on features (or feature sets).

It appears that the meaning representation envisioned by fuzzy sets is holistic in nature trying to load all meaning for graded membership to a single mathematical function called the membership function. The featural theories also found their way into fuzzy set theory using the concepts of conjoint measurement [19], an area that still has room for development.

However, other models of word meaning representation (network models, semantic fields, and computational and statistical linguistic models) also require academic attention in connection with the representation of degrees of membership.

7.3 Representational Theory of Measurement

We now turn to the question of measurement of the membership function. Without delving into the practical measurement issues we discuss the implications of such a measurement. To put this question into perspective we resort to the representational theory of measurement (RTM) [8, 14]. RTM postulates axioms under which measurement (defined as a mapping from a qualitative algebraic structure to a numerical structure) is possible (representation) and meaningful. The meaningfulness comes as a result of the concept of a scale which is related to the uniqueness of the representation. If the representation is unique up to a positive linear transformation the resulting scale is called an interval scale where ordering and averaging are meaningful whereas if the representation is unique up to a similarity transformation the resulting scale is a ratio scale where not only ordering and averaging but taking ratios are all meaningful. This elaborate theory of measurement is the dominant view about how we think about measurement today.

The concept of meaningfulness prescribed by the RTM and the resulting scale types can shed light into the semantics of fuzzy set theory. Measurement of membership functions has been investigated in the context of RTM in quite a few studies [2, 4, 11, 12, 16, 19, 23]. What the RTM entails for measuring the membership function is an objective interpretation of fuzziness.

There are three main objections to the RTM [5, 13]: (i) RTM cannot be applied because it uses an axiomatic approach, and because it cannot give a satisfactory account of actual scientific measurement practice, (ii) Errors of measurement cannot be incorporated into the RTM framework, (iii) RTM is wrongly liberal in what it accepts as measurement. Although we do not agree with the first of these criticisms, the other two are largely accepted and partially defended by the proponents of RTM [10, 15].

2 In fact, RTM carefully insists on constructive proofs of representation theorems. Such a constructive approach is expected to lead to practical measurement concepts like standard sequences [8].