In the previous chapter, we focused our attention on static electric charges that are fixed in space and constant in time. Otherwise, we assumed that the charges relax to a steady distribution in an instant. Electric charges, however, can move under the influence of an electric field. The charges moving in a conductor constitute a conduction current, while those moving in a vacuum constitute a convection current. From basic circuit theory, the readers should be familiar with the conduction current flowing in a simple electric circuit, which is governed by Ohm’s law, stating that the voltage across a resistor is equal to the product of the resistance and the current passing through it. According to the principle of conservation of charge, electric charges cannot be created or destroyed. This principle manifests itself as the equation of continuity in electromagnetics, and Kirchhoff’s current law in circuit theory, stating that the sum of all currents entering a junction in an electric circuit is equal to zero. On a macroscopic scale, when we are concerned with the currents flowing in conducting wires, the current is defined as charges passing through a reference point per unit time. At the microscopic scale, when the magnitude and direction of the current are assumed to vary as functions of position in a region of space, we define the current density as charges passing through a reference point per unit area per unit time.

In electromagnetics, we frequently encounter three types of currents: conduction current, convection current, and displacement current density. In the conductor, the loosely bound valence electrons easily detach themselves from the host atoms and make up a sea of electrons, called free electrons or conduction electrons. In the presence of an externally applied electric field, the free electrons gain an average velocity called a drift velocity, and form a conduction current. In contrast, charged particles moving in a vacuum or in a rarefied gas constitute a convection current. The electron beam in a cathode-ray tube, the accelerated electrons in a photomultiplier, and the electrical discharge in a bolt of lightning are a few examples of convection currents. While the conduction and convection currents are directly related to the motion of electric charges, the displacement current density is an equivalent current that involves no electric charges, but behaves as a conduction or convection current as far as the time-varying magnetic field is concerned.
4.1 Convection Current

The convection current is formed by the electric charges moving in a vacuum. In order to describe the spatial variation of the current, we define the current density as charges passing through a unit area of the cross section per unit time. In view of the definition, the current density can be regarded as a kind of flux density. The current density is measured in amperes per square meter \([A/m^2]\), or coulombs per square meter per second \([C/m^2\cdot sec]\). While the current is a scalar quantity, the current density is a vector quantity, which may vary from point to point in space, forming a vector field in a region of space. It is important to note that the current density is defined as the current through a cross section, or a plane perpendicular to the direction of the current.

Let us consider Fig. 4.1, in which a volume charge of an uniform density \(\rho_v [C/m^3]\) moves with a constant velocity \(\mathbf{v}\), passing through a surface \(\mathcal{S}\). The total charge crossing an incremental area \(\Delta s\) in a short period of time \(\Delta t\) is

\[
\Delta Q = \rho_v \mathbf{v} |\Delta t \Delta s \cos \theta|
\]  

(4-1)

In the above equation the term \(\Delta s \cos \theta\) represents the equivalent area of \(\Delta s\), which is given in the cross section, or the projection of \(\Delta s\) onto the plane perpendicular to the direction of \(\mathbf{v}\). Upon using the relation \(\cos \theta = \mathbf{a}_v \cdot \mathbf{a}_s\), where \(\mathbf{a}_v\) is a unit vector in the direction of \(\mathbf{v}\) and \(\mathbf{a}_s\) is a unit normal to \(\Delta s\), Eq. (4-1) becomes

\[
\Delta Q = \rho_v \mathbf{v} |\Delta t \Delta s (\mathbf{a}_v \cdot \mathbf{a}_s) = \rho_v \mathbf{v} \cdot \Delta s \Delta t
\]

(4-2)

where we used \(\mathbf{v} = |\mathbf{v}| \mathbf{a}_v\) and \(\Delta s = \Delta s \mathbf{a}_s\). The incremental current through the incremental area \(\Delta s\) is therefore

\[
\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \mathbf{v} \cdot \Delta s = \mathbf{J} \cdot \Delta s
\]

(4-3)

The current density \(\mathbf{J}\) is defined, from Eq. (4-3), as

\[
\mathbf{J} \equiv \rho_v \mathbf{v} [A/m^2]
\]

(4-4)

where \(\rho_v\) is the volume charge density, and \(\mathbf{v}\) is the velocity of flow of the charges. The current density has the unit of the ampere per square meter \([A/m^2]\). The current density is a vector whose unit vector points in the direction of flow of the current, and whose magnitude equals the charges crossing a unit area of the