The Life of π: From Archimedes to ENIAC and Beyond

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Abstract The desire to understand π, the challenge, and originally the need, to calculate ever more accurate values of π, the ratio of the circumference of a circle to its diameter, has captured mathematicians — great and less great — for many centuries. And, especially recently, π has provided compelling examples of computational mathematics. π, uniquely in mathematics, is pervasive in popular culture and the popular imagination. In this paper, I intersperse a largely chronological account of π's mathematical and numerical status with examples of its ubiquity.

Preamble: π and Popular Culture

The desire to understand π, the challenge, and originally the need, to calculate ever more accurate values of π, the ratio of the circumference of a circle to its diameter, has challenged mathematicians — great and less great — for many centuries and, especially recently, π has provided compelling examples of computational mathematics. π, uniquely in mathematics, is pervasive in popular culture and the popular imagination.[1]

I shall intersperse this largely chronological account of π's mathematical status with examples of its ubiquity. More details will be found in the selected references at the end of the chapter — especially in *Pi: A Source Book* [Berggren, Borwein and Borwein 2004]. In Berggren, Borwein and Borwein [2004] all material not otherwise referenced may be followed up, as may much other material, both serious and fanciful. Other interesting material is to be found in Eymard and Lafon [2003], which includes attractive discussions of topics such as continued fractions and elliptic integrals.

Fascination with π is evidenced by the many recent popular books, television shows, and movies — even perfume — that have mentioned π. In the 1967 *Star Trek* episode "Wolf in the Fold," Kirk asks, "Aren't there some mathematical problems that simply can't be solved?" And Spock "fries the brains" of a rogue computer by telling it, "Compute to the last digit the value of π." The May 6, 1993 episode of The Simpsons has the character Apu boast, "I can recite π to 40,000 places. The last digit is one." (See Figure 1.)

In November 1996, MSNBC aired a Thanksgiving Day segment about π, including that scene from Star Trek and interviews with the present author and several other mathematicians at Simon Fraser University. The 1997 movie *Contact*, starring Jodie Foster, was based on the 1986 novel by noted astronomer Carl Sagan. In the book, the lead character searched for patterns in the digits of π, and after her mysterious experience found sound confirmation in the base-11 expansion of π. The 1997 book *The Joy of Pi* [Blatner 1997] has sold many thousands of copies and continues to sell well. The

This paper is an updated and revised version of Borwein [2008], and is made with permission of the editor.

[1] The *MacTutor* website, http://turnbull.mcs.st-and.ac.uk/history/, at the University of St. Andrews — my home town in Scotland — is rather a good accessible source for informal mathematical history.

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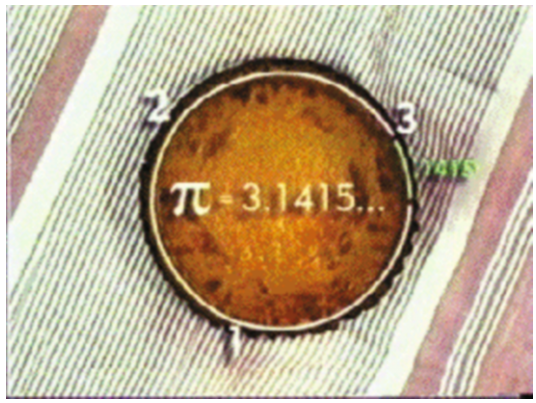

Figure 1: Around 250 BCE, Archimedes of Syracuse (287–212 BCE) was the first to show that the "two possible π's" are the same. Clearly for a circle of radius r and diameter d, **Area**$= \pi_1 r^2$ while **Perimeter** $= \pi_2 d$, but that $\pi_1 = \pi_2$ is not obvious, and is often overlooked (see Fig. 4). Courtesy of Guilio Einaudi Editori.

1998 movie entitled *Pi* began with decimal digits of π displayed on the screen. And in the 2003 movie *Matrix Reloaded*, the Key Maker warns that a door will be accessible for exactly 314 seconds, a number that *Time* speculated was a reference to π.

As a forceable example, imagine the following excerpt from Eli Mandel's 2002 Booker Prize winning novel *Life of Pi* being written about another transcendental number:

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My name is
Piscine Molitor Patel
known to all as Pi Patel.
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For good measure I added

$$\pi = 3.14$$

and I then drew a large circle which I sliced in two with a diameter, to evoke that basic lesson of geometry.

Equally, National Public Radio reported on April 12, 2003 that novelty automatic teller machine withdrawal slips, showing a balance of $\$314,159.26$, were hot in New York City. One could jot a note on the back and, apparently innocently, let the intended target be impressed by one's healthy savings account. Scott Simon, the host, noted the close resemblance to π. Correspondingly, according to the *New York Times* of August 18 2005, Google offered exactly "14,159,265 New Slices of Rich Technology" as the number of shares in its then new stock offering. Likewise, March 14 in North America has become π *Day*, since in the USA the month is written before the day (314). In schools throughout North America, it has become a reason for mathematics projects, especially focussing on π.

As another sign of true legitimacy, on March 14, 2007 the *New York Times* published a crossword in which to solve the puzzle, one had first to note that the clue for 28 DOWN was "March 14, to Mathematicians," to which the answer is PIDAY. Moreover, roughly a dozen other characters in the puzzle are PI — for example, the clue for 5 DOWN was "More pleased" with the six character answer HAPπER. The puzzle is reproduced in Borwein and Bailey [2008].

It is hard to imagine e, γ or $\log 2$ playing the same role. A corresponding scientific example [von Baeyer 2003, 11] is

> A coded message, for example, might represent gibberish to one person and valuable information to another. Consider the number 14159265... Depending on your prior knowledge, or lack thereof, it is either a meaningless random sequence of digits, or else the fractional part of π, an important piece of scientific information.