Testing the Foundations of Quantal Response Equilibrium

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Abstract. Quantal response equilibrium (QRE) has become a popular alternative to the standard Nash equilibrium concept in game theoretic applications. It is well known that human subjects do not regularly choose Nash equilibrium strategies. It has been hypothesized that subjects are limited by strategic uncertainty or that subjects have broader social preferences over the outcome of games. These two factors, among others, make subjects boundedly-rational. QRE, in essence, adds a logistic error function to the strict, knife-edge predictions of Nash equilibria. What makes QRE appealing, however, also makes it very difficult to test, because almost any observed behavior may be consistent with different parameterizations of the error function. We present the first steps of a research program designed to strip away the underlying causes of the strategic errors thought to be modeled by QRE. If these causes of strategic error are correct explanations for the deviations, then their removal should enable subjects to choose Nash equilibrium strategies. We find, however, that subjects continue to deviate from predictions even when the reasons presumed by QRE are removed. Moreover, the deviations are different for each and every game, and thus QRE would require the same subjects to have different error parameterizations. While we need more expansive testing of the various causes of strategic error, in our judgment, therefore, QRE is not useful at predicting human behavior, and is of limited use in explaining human behavior across even a small range of similar decisions.

Keywords: bounded rationality, human behavior, Nash equilibrium, behavioral game theory, strategic uncertainty, social preferences, Quantal Response Equilibrium.

1 Game Theoretic Models of Behavior

A common approach to predicting strategic human behavior across many different situations utilizes the theory of non-cooperative games. At the same time, decades worth of human subject experiments now demonstrate that the basic predictions of game theory are inaccurate [1, 2, 3]. As a result, scholars have developed a variety of modifications of the standard Nash equilibrium concept that relax the strict behavioral and cognitive assumptions contained in Nash equilibrium.
One of the most well-known models that relaxes the predictions of Nash equilibrium is the Quantal Response Equilibrium (QRE) [4]. Some argue that QRE “almost always explains the direction of deviations from Nash and should replace Nash as the static benchmark to which other models are routinely compared.” [5] The QRE model maintains the assumption that individuals have beliefs that are supported in equilibrium by the strategies that players choose, but that players make systematic “mistakes” or deviations in their choices. The individual deviations from Nash equilibrium strategies and outcomes can come from a variety of different sources, but two reasons seem to predominate discussions. The first reason is strategic uncertainty or bounded rationality [6]. The second reason is that individual utility functions may involve something other than the individual subject’s payoffs, typically thought of as a “social preference,” in which subjects appear altruistic or fair or seek to reciprocate fairness or seek to limit inequality in payoffs [7, 8].

In the QRE model the deviations between subjects’ actual behavior and the Nash equilibrium of a game are represented by a parameter, $\lambda$, and as $\lambda$ approaches zero, behavior is essentially random, while as $\lambda$ approaches infinity, behavior is consistent with the strategies predicted by Nash equilibrium. The parameter in QRE is a free parameter that is fitted via maximum likelihood estimation and is used to fit the model to the observed behavior. The source or interpretation of the parameter, however, is not determined a priori but is left to the analyst. The QRE model predicts that deviations from Nash predictions will be less likely as behavior becomes more costly in terms of the losses a player suffers from non-equilibrium (non-rational) behavior.

The QRE model has been widely demonstrated to provide a better fit to observed behavior than Nash equilibrium, and has become popular in fields outside of economics, such as computer science. The most rigorous statistical examinations of QRE estimate $\lambda$ from a subset of experimental data and then examine how the estimated $\lambda$ “predicts” the remaining, unused or out-of-sample data (basically a form of cross-validation or training/test data) [9]. While clever and exciting, this approach falls short as a test of the QRE model for two main reasons.

First, this approach does not make specific predictions about expected behavior prior to the design of the study. That is, the QRE approach does not rule out much, if any, behavior as inconsistent with the theory, data is collected and used to estimate a free parameter that can accommodate almost any observations. Despite the empirical failure of the Nash-equilibrium model, it has the virtue that it does make clear predictions about the expected pattern of observed behavior. To put it another way, is there a distribution of observed behavior that would suggest QRE is an incorrect model of behavior?

Second, statistical approaches to QRE take experimental data and then identify which model best “fits” (by some standard) that data. It is unspecified how to reject QRE in this approach. We could certainly find that some other model better fits the data or that combining QRE with another model leads to a better overall fit to the distribution of the data [9]; however, this approach does not reject the model.

An alternative approach to testing QRE is to identify the underlying source of the deviations from rationality (Nash equilibrium strategies) that give rise to $\lambda$, and then design experiments that remove these sources and observe whether deviations still