8.1 Introduction

In recent years, increasing research interest has been paid to the study of networked control systems (NCSs) due to the fact that compared with the traditional point-to-point communication, NCSs have several advantages such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. It is well known that in NCSs, the network-induced time delays (communication delays) and missing measurements (packet dropouts) are always inevitable that would degrade the system performance or even cause instability. It should be pointed out that in most results of the relevant literature, the network-induced time delays are assumed to be constant. Although the ideal assumption can simplify the analysis and synthesis of NCSs, it often cannot be satisfied because delays resulting from network transmissions are typically time-varying and random. Thus, more and more attention has been paid to model the network-induced time delays in various time-varying and probabilistic ways. It should be pointed out that up to now, most proposed results concerning missing measurements are based on an assumption that the measurement signal is either completely missing or completely available, and all the sensors have the same data missing probability. Such an assumption, however, inevitably limits the scope of applications of the given results, since it cannot be applied to some practical cases where multiple missing measurements occur, for example, the case when only partial information is missing and the case when the individual sensor has different missing probability. Very recently, a new way to describe the missing measurements has been proposed in the literature, where the missing probability for each sensor is governed by an individual random variable satisfying a certain probabilistic distribution over the interval [0, 1]. It is clear that the description of missing measurements in the literature is much more general and desirable than that in the previous literature.
In this chapter, the problem of $H_\infty$ filtering is investigated for networked discrete-time SSs by using LMI method. The effects of multiple probabilistic time-varying communication delays and multiple missing measurements on the performance of the filtering error system are considered. The derived criteria for performance analysis of the filtering error systems and filter design are formulated by LMI. A numerical example shows the effectiveness of the design method.

8.2 Problem Formulation

Consider the following networked discrete-time SS with multiple stochastic communication delays:

\begin{equation}
\begin{cases}
E x(k+1) = A x(k) + A_d \sum_{i=1}^{m} \alpha_i(k) x(k - d_i(k)) \\
+ A_\tau \sum_{j=1}^{s} \beta_j(k) \sum_{v=1}^{\tau_j(k)} x(k - v) + B_\omega \omega(k), \\
z(k) = L x(k),
\end{cases}
\end{equation}

where $x(k) \in \mathbb{R}^n$ is the state, $z(k) \in \mathbb{R}^q$ is the signal to be estimated, and $\omega(k) \in \mathbb{R}^p$ is the disturbance input that belongs to $l_2[0, \infty)$. $E$, $A$, $A_d$, $A_\tau$, $B_\omega$, and $L$ are known real constant matrices with appropriate dimensions. The matrix $E$ may be singular and it is assumed that $\text{rank } E = r \leq n$. $d_i(k)$ $(i = 1, 2, \cdots, m)$ denote the discrete time-varying delays satisfying

\begin{equation}
d_i^1 \leq d_i(k) \leq d_i^2,
\end{equation}

where $d_i^1$ and $d_i^2$ are constant positive integers representing the lower and upper bounds on the discrete delays, respectively. $\tau_j(k)$ $(j = 1, 2, \cdots, s)$ denote the distributed time-varying delays satisfying

\begin{equation}
\tau_1^j \leq \tau_j(k) \leq \tau_2^j,
\end{equation}

where $\tau_1^j$ and $\tau_2^j$ are constant positive integers representing the lower and upper bounds on the distributed delays, respectively.

Remark 8.1. The communication delays considered in (8.1) are first introduced in [24, 25]. This type of delay widely exists in NCSs. However, SSs with such type of communication delays have not been fully considered.

The random variables $\alpha_i(k)$ $(i = 1, 2, \cdots, m)$ and $\beta_j(k)$ $(j = 1, 2, \cdots, s)$ in (8.1) are mutually uncorrelated Bernoulli distributed white sequences obeying the following probability distribution law [24]:

\begin{equation}
\begin{cases}
\text{Pr}\{\alpha_i(k) = 1\} = \delta\{\alpha_i(k)\} = \bar{\alpha}_i, \text{Pr}\{\alpha_i(k) = 0\} = 1 - \bar{\alpha}_i, \\
\text{Pr}\{\beta_j(k) = 1\} = \delta\{\beta_j(k)\} = \bar{\beta}_j, \text{Pr}\{\beta_j(k) = 0\} = 1 - \bar{\beta}_j.
\end{cases}
\end{equation}