Deterministic Logics for UL

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Abstract. The class of Unambiguous Star-Free Regular Languages (UL) has been widely studied and variously characterized by logics such as $TL[X_a,Y_a]$, $UITL$, $TL[F,P]$, $FO^2[\prec]$, the variety DA and partially-ordered two-way DFA. However, explicit reductions from logics to automata are missing. In this paper, we introduce the concept of Deterministic Logics for UL. The formulas of deterministic logics uniquely parse a word in order to evaluate satisfaction. We consider three such deterministic logics with varied modalities, namely $TL[X_a,Y_a]$, $TL[\tilde{U},\tilde{S}]$ and $UITL^\pm$. Using effective reductions between them and to $po2dfa$, we show that they all characterize UL, and have NP-complete satisfiability. The reductions rely on features of deterministic logic such as unique parsability and ranker-directionality.

1 Introduction

Unambiguous star-free regular languages (UL) was a language class first studied by Schützenberger [Sch76]. He gave an algebraic characterization for UL using the monoid variety DA. Since then, several diverse and unexpected characterizations have emerged for this language class: $\Delta_2[\prec]$ in the quantifier-alternation hierarchy of first-order definable languages [EPW97], the two variable fragment $FO^2[\prec]$ [TW98] (without any restriction on quantifier alternation), and Unary Temporal Logic $TL[F,P]$ [EVW02] are some of the logical characterizations that are well known. Investigating the automata for UL, Schwentik, Therien and Volmer [STV01] defined Partially Ordered 2-Way Deterministic Automata ($po2dfa$) and showed that these exactly recognize the language class UL. Recently, there have been additional characterizations of UL using deterministic logics $UITL$ [LPS08] as well as $TL[X_a,Y_a]$ [DK07]. A survey paper [DGK08] describes this language class and its characterizations.

A monomial over an alphabet $\Sigma$ is a regular expression of the form $A_0^*a_1 \cdots a_{n-1}^*A_n^*$, where $A_i \subseteq \Sigma$ and $a_i \in \Sigma$. By definition, UL is the subclass of star-free regular languages which may be expressed as a finite disjoint union of unambiguous monomials: every word that belongs to the language, may be unambiguously parsed so as to match a monomial. The uniqueness with which these monomials parse any word is the characteristic property of this language class. We explore a similar phenomenon in logics by introducing the notion of Deterministic Temporal Logics for UL.

Given a modality $\mathcal{M}$ of a temporal logic that is interpreted over a word model, the accessibility relation of $\mathcal{M}$ is a relation which maps every position in the word with the set of positions that are accessible by $\mathcal{M}$. In case of interval temporal logics, the relation is over intervals instead of positions in the word model. The modality is deterministic if its accessibility relation is a (partial) function. A logic is said to be deterministic if all its
modalities are deterministic. Hence, deterministic logics over words have the property of Unique Parsability.

**Definition 1 (Unique Parsability).** In the evaluation of a temporal logic formula over a given word, every subformula has a unique position (or interval) in the word at which it must be evaluated. This position is determined by the context of the subformula.

In this paper we relate three deterministic temporal logics and investigate their properties. We give constructive reductions between them (as depicted in Figure 1) and also to the $po2dfa$ automata. Hence, we are able to infer their expressive equivalence with the language class $UL$. Moreover, the automaton connection allows us to establish their NP-complete satisfiability.

(i) Deterministic Until-Since Logic - $TL[\tilde{U}, \tilde{S}]$:
Let $A$ be any subset of the alphabet and $b$ be any letter from the alphabet. The "deterministic half until" modality $A\tilde{U}_b\phi$ holds if at the first occurrence of $b$ in (strict) future $\phi$ holds and all intermediate letters are in $A$. The past operator $A\tilde{S}_b\phi$ is symmetric. Since the modalities are deterministic, the formulas possess the property of unique parsability. This logic admits a straightforward encoding of $po2dfa$.

(ii) Unambiguous Interval Temporal Logic with Expanding Modalities - $UITL^\pm$:
This is an interval temporal logic with deterministic chop modalities $F_a$ and $L_a$ which chop an interval into two at the first or last occurrence of letter $a$. These modalities were introduced in [LPS08] as logic $UITL$. Here, we enrich $UITL$ with the expanding $F_a^+$ and $L_a^-$ chop modalities that extend an interval beyond the interval boundaries in the forward and the backward directions to the next or the previous occurrence of $a$. We call this logic $UITL^\pm$. It is a deterministic logic.

(iii) Deterministic Temporal Logic of Rankers - $TL[X_a, Y_a]$:
Modality $X_a\phi$ (or $Y_a\phi$) accesses the position of the next (or the last) occurrence of letter $a$ where $\phi$ must hold. The temporal logic with these modalities was investigated in [DK07]. The authors showed that the deterministic temporal logic $TL[X_a, Y_a]$ which closes the rankers of [WI07, STV01] under boolean operations, characterizes $UL$ (their work was in the setting of infinite words). We identify $TL[X_a, Y_a]$ as a deterministic logic and use its property of unique parsability to give an efficient reduction from formulas to $po2dfa$.

![Fig. 1. Deterministic Logics and po2dfa with reductions as presented in this paper](image-url)