Background Recovery by Fixed-Rank Robust Principal Component Analysis

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Abstract. Background recovery is a very important theme in computer vision applications. Recent research shows that robust principal component analysis (RPCA) is a promising approach for solving problems such as noise removal, video background modeling, and removal of shadows and specularity. RPCA utilizes the fact that the background is common in multiple views of a scene, and attempts to decompose the data matrix constructed from input images into a low-rank matrix and a sparse matrix. This is possible if the sparse matrix is sufficiently sparse, which may not be true in computer vision applications. Moreover, algorithmic parameters need to be fine tuned to yield accurate results. This paper proposes a fixed-rank RPCA algorithm for solving background recovering problems whose low-rank matrices have known ranks. Comprehensive tests show that, by fixing the rank of the low-rank matrix to a known value, the fixed-rank algorithm produces more reliable and accurate results than existing low-rank RPCA algorithm.

Keywords: Background recovery, reflection removal, robust PCA.

1 Introduction

Background recovery is a very important recurring theme in computer vision applications. Traditionally, different approaches have been developed to solve different varieties of the problem. Recent research in robust principal component analysis (RPCA) offers a promising alternative approach for solving problems such as noise removal, video background modeling, and removal of shadows and specularity [2][12]. RPCA utilizes the fact that multiple views of a scene contain consistent information about the common background. It constructs a data matrix from multiple views and decomposes it into a low-rank matrix that contains the background and a sparse matrix that captures non-background components. It has been proved that exact solution of RPCA problem is available if the data matrix is composed of a sufficiently low-rank matrix and a sufficiently sparse matrix [2][3][9][13]. Various algorithms have been proposed for solving RPCA problem [6][9][12]. In particular, the methods based on augmented Lagrange multiplier (ALM) have been shown to be among the most efficient and accurate methods [0].
In computer vision applications, the non-background components may not be sparse. Moreover, algorithmic parameters need to be fine tuned to yield accurate results [1]. These difficulties are especially pronounced for reflection removal problem, and no work on applying RPCA to reflection removal has been reported so far. Fortunately, these application problems can be framed as one of recovering a fixed-rank matrix from the data matrix because the rank of the low-rank matrix is known. This paper proposes a fixed-rank RPCA algorithm based on ALM (FrALM) for solving background recovering problems. Comprehensive tests on reflection removal and video background modeling show that FrALM produces more accurate results than does low-rank ALM method (LrALM). Moreover, FrALM can produce optimal or near optimal results over a much wider range of parameter values than does LrALM, making it more reliable for solving computer vision problems whose low-rank matrices have known ranks.

2 Existing RPCA Methods

Robust PCA is a term given to a long line of work that aims to render PCA robust to gross corruption and outliers. Various methods have been proposed including influence function [4], multivariate trimming [7], alternating minimization [8], and random sampling [5]. These methods are either inefficient, having non-polynomial time complexity, or do not guarantee optimal solutions [12].

A recent approach directly decomposes a corrupted data matrix into a low-rank matrix and a sparse matrix. The corruption is assumed to be sparse, but the noise amplitude can be large. Various methods have been proposed such as iterative thresholding [9], proximal gradient [12], accelerated proximal gradient [6], and augmented Lagrange multiplier method (ALM) [9]. In particular, ALM has been shown to be among the most efficient and accurate methods [9]. These methods require tuning of algorithmic parameters [1]. On the other hand, [1] applies Bayesian approach to estimate the algorithmic parameters along with the matrices based on prior distributions of inverse variances.

In our applications, the rank of the low-rank matrix is known. So, we adopt the ALM approach but fix the rank of the low-rank matrix, which provides more specific constraint than do prior distributions. This approach allows our algorithm to converge efficiently and accurately, as for the low-rank ALM method of [9], and is simpler and more efficient than the Bayesian method of [1].

Other methods have been proposed to solve related but different problems. For example, [11] solves low-rank matrix factorization and [10] computes a fixed-rank representation for sparse subspace clustering. They are not directly applicable to our application problem, which is a matrix decomposition problem.

3 Fixed-Rank RPCA

Given an \( m \times n \) data matrix \( D \), PCA seeks to recover a low-rank matrix \( A \) from data matrix \( D \) such that the discrepancy or error \( E = D - A \) is minimized:

\[
\min_{A,E} \|E\|_F, \text{ subject to } \text{rank}(A) \leq r, \ D = A + E
\]

(1)