In this article, I describe alternative types of optimization models for minimizing distribution costs. After reviewing models for the case of deterministic demand, several applications are described which illustrate the cost savings that can be attained with such models. Extensions to the case of stochastic demand are then given. Alternative models for distribution problems with stochastic demand are described, and the implications on computational difficulty of these different types of models are discussed.

Introduction

Distribution planning is an important problem faced by many organizations today, due to the expense of transporting goods from factories to customers. In the United States alone, the annual cost of transporting goods exceeded $100 billion during the early 1970's [Sampson and Farris, 1975]. As energy costs have risen, the expense of transporting goods has grown accordingly, thus increasing the importance of distribution planning. Furthermore, with the advent of deregulation of transportation industries in the U.S., freight rates have been changing much more frequently than in the past, making the problem of minimizing distribution costs all the more difficult to solve by informal "rule-of-thumb" approaches. While the cost of transportation has increased significantly and rate changes have become more frequent, the cost of computers for solving these problems has dropped very rapidly. This changing cost structure, along with the realization of the effect on profit of more precise scheduling of distribution, have stimulated the use of rigorous analytical models for solving distribution problems. In the next sections, I review analytical models for distribution planning and discuss the computational requirements of solving these models.

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Deterministic Distribution Models

The classical Hitchcock Transportation Model is to choose shipment quantities of a single typical product from factory areas (or other supply points) to customer areas (or other demand points) to minimize shipping costs, while recognizing factory capacities and customer requirements. This model assumes that shipping costs are proportional to the amounts shipped. The following notation is used to describe this model:

- \( c_{ij} \) = cost per unit shipped from factory \( i \) to customer area \( j \)
- \( S_i \) = supply or capacity at factory \( i \) in each time period
- \( D_j \) = demand by customer area \( j \) in each time period
- \( x_{ij} \) = amount shipped from factory \( i \) to customer area \( j \) in each time period (decision variable).
- \( m \) = number of factories
- \( n \) = number of customer areas

The transportation model is

\[
\text{MIN} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \text{(shipping cost)} \quad (1)
\]

\text{SUBJECT TO}

\[
\sum_{j=1}^{n} x_{ij} \leq S_i \quad i = 1, 2, \ldots, m \quad \text{(factory capacities)} \quad (2)
\]

\[
\sum_{i=1}^{m} x_{ij} \geq D_j \quad j = 1, 2, \ldots, n \quad \text{(customer requirements)} \quad (3)
\]

\[
x_{ij} > 0 \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n \quad \text{(4)}
\]

The transportation model can be used to answer "what-if" questions, such as "What if there is a rate change from factory 2 to customer area 4?" or "What if customer demand increases by 20%?" or "What if we build a new factory near customer area 1 - would shipping costs decrease sufficiently to justify this investment?" One of the principal advantages of a model such as (1) - (4) is that once it is