Stochastic Optimization Approach to Dynamic Problems with Jump Changing Structure

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We consider dynamic optimization models the structure of which (functional, equations, constrains) can be changed at time depending on control strategy. The main problem is a choice of optimal structure and a strategy which provides an optimal transition from one structure to another.

The problem under discussion is connected with modelling global changes in economic mechanism (for example, a transition from Centralization to Market), radical technological innovations and so on.

In deterministic case these problems are nonconvex and nonsmooth. We propose general approach to such models based on their stochastic approximations and obtain a stochastic programming problem with controlled measure.

This approach is illustrated by economic dynamic model with endogenous innovations.

1 General Economic Dynamic Model

We study the general multi-sector economic dynamic model with discretely expanding technologies. Emergence moments of new technological modes (new technological structure) are defined by given levels of expenditure on R&D (research and development) and by the strategy of investments into R&D. The model may be written in the following form:

\[
\sum_{k=0}^{N} \sum_{t=\theta_k}^{\theta_{k+1}} \varphi_k(a_t, b_{t+1}) \Rightarrow \max, \\
(a_t, b_{t+1}) \in T_k, \quad \theta_k \leq t < \theta_{k+1}, \quad (c_t, d_{t+1}) \in Q_k, \quad b_t \geq c_t + a_t \\
\theta_k = \min\{t : \sum_{j=\theta_{k-1}}^{t} d_j \geq \xi_k\}, \quad d_{\theta_k} = 0, k = 1, \ldots, N, \quad \theta_{N+1} = \tau - 1,
\]

where convex technological set \( T_k \) (i.e. a set of 'input–output' vectors \((a, b)\)) and concave utility function \( \varphi_k \) form structure of economic system; vector \( \xi_k \) is the level of expenditures necessary for the transition from the structure

\((\varphi_k, T_k)\) to \((\varphi_{k+1}, T_{k+1})\); \(\theta_k\) is the corresponding transition moment, and convex set \(Q_k\) specifies dynamics of assets in R&D for unit interval.

This model is nonconvex in general. We use a stochastic version of the initial model, which takes into account an uncertainty of expenditure levels on R&D and incompleteness of information on parameters of future technologies. The stochastic model is already locally convex. This fact allows us to formulate the stochastic maximum principle for the model and to find the optimal structure as well as optimal strategy of transition.

For simple case \((N = 1)\) this problem was study in [1]. Model of structural transition from centralized economy to market economy was considered in [2]. Analysis of the above model essentially use the following generalization of stochastic maximum principle from [3].

2 Stochastic Maximum Principle with Controlled Measure

Let \(\{\eta_t, t = 0, 1, \ldots, \tau\}\) be stochastic process with values in measurable space \((S, E)\) with transition functions \(\Pi^{t+1}(\eta^t, x_t, u_t, d\eta_{t+1})\), depending measurably on the process \(x_t \in \mathbb{R}^n\), and on control \(u_t \in U\), where \(U\) is Polish space, \(\eta^T = (\eta_0, \ldots, \eta_T), \tau < \infty\). The process \(x_t\) is described by the system of difference equations:

\[
x_{t+1} = f^{t+1}(\eta^{t+1}, x_t, u_t), \quad x_0 = x_0(\eta_0)
\]  

(1)

Suppose that measures \(\Pi^{t+1}\) are absolutely continuous with respect to some (fixed) transition measure \(\pi^{t+1}(\eta^t, x_t, u_t, \eta_{t+1})\); \(\pi^{t+1}(\eta^t, x_t, u_t, \eta_{t+1})\) is density with respect to \(\eta_{t+1}\). Each control \(u_t = u_t(\eta^t)\) generates a measure on the space of sequences \(\{\eta^t\}\). It is required to maximize the functional:

\[
E^u \sum_{0}^{\tau-1} \tilde{\mathcal{F}}^t(\eta^t, x_t, u_t)
\]  

(2)

subject to restrictions

\[
E^u \sum_{0}^{\tau-1} \tilde{\varphi}(\eta^t, x_t, u_t) \geq 0, \quad g(\eta^t, x_t, u_t) \leq 0 \quad (u_t \in U), \quad (P - a.s.),
\]

(3)

where \(P\) is a measure generated by the initial distribution \(P_0(d\eta_0)\) and the transition function \(P^{t+1}(\eta^t, d\eta_{t+1})\), \(\tilde{\mathcal{F}}^t(\cdot) \in \mathbb{R}^1\), \(\tilde{\varphi}^t(\cdot) \in \mathbb{R}^m\), \(g^t(\cdot) \in \mathbb{R}^k\). Let denote