1. Introduction

Neuron models have been investigated from various points of view. One of the most fundamental viewpoints is to study the response characteristic of the neuron model to the periodic pulse input.

Based on the experiments on an analog neuron model using transistors, Harmon [1] first demonstrated that the relation between the amplitude of the input pulse and the firing rate of the neuron model has a very complicated form as shown in Fig. 1.

In order to explain the result, Nagumo and Sato [2] studied the following Caianiello nerve equation:

\[ x_{n+1} = 1[\beta - \alpha \sum_{r=0}^{n} c^r x_{n-r} - \theta], \]  

(1.1)

where \(1[z] = 1\) for \(z > 0\), and \(= 0\) for \(z \leq 0\). They showed that the relation between the amplitude of the input pulse and the firing rate is an extended Cantor function. By the use of new variable

\[ y_n = \sum_{r=0}^{n} c^r x_{n-r}, \]  

(1.2)

(1.1) is rewritten into the following difference equation:

\[ y_{n+1} = Cy_n + \lambda \quad \text{for} \ y_n \leq 0, \]  

\[ y_{n+1} = Cy_n - (1 - \lambda) \quad \text{for} \ y_n > 0, \]  

(1.3)

where \(\lambda = (\beta - \theta)/\alpha\) represents the input to the neuron model.
Hata [3] studied (1.3) mathematically and showed that (1.3) has non-periodic output sequence if and only if \( \lambda \) belongs to the exceptional set whose measure is zero.

With the intention of showing the similar result for more realistic neuron models, we investigate the BVP model which was proposed by FitzHugh [4] and Nagumo et al. [5] as a simplified realization of the Hodgkin-Huxley equation. First, we consider the BVP model with a simple nonlinear characteristic and then we treat a more general case.

The results obtained in this paper will be applicable to the study of the entrainment problem such as arrhythmias.

Fig.1 Response characteristic of Harmon's electronic neuron model. The relation between the pulse amplitude of the stimulating pulse sequence with a fixed frequency (abscissa) and the firing rate of the stimulated neuron model (ordinate).