CHAPTER I. SOLOW MODEL

1. Fixed Deficit Per Head

1.1. Short-Run Equilibrium

The research will be carried out within the following framework. Firms manu­facture a homogeneous commodity by employing capital and labour. Let the production function be of the Cobb-Douglas variety $Y = K^\alpha N^\beta$. Output can be devoted to consumption, investment, government purchases and net exports $Y = C + I + G + H$. Let labour grow at the natural rate $\dot{N} = nN$ with $n = \text{const}$. For the small open economy, the foreign interest rate is given $r^* = \text{const}$. Under perfect capital mobility, the domestic interest rate coincides with the foreign interest rate $r = r^*$. Firms maximize profits under perfect competition, so the marginal product of capital agrees with the interest rate $\alpha Y/K = r$. This in turn yields the stock of capital. By way of contrast, the wage rate harmonizes with the marginal product of labour $w = \beta Y/N$.

Now have a look at capital dynamics. $Y = K^\alpha N^\beta$, $\alpha Y/K = r$ and $\dot{N} = nN$ can be reformulated in terms of growth rates as $\dot{Y} = \alpha \dot{K} + \beta \dot{N}$, $\dot{K} = \dot{Y}$ and $\dot{N} = n$, where the hat symbolizes the growth rate. From this one can infer $\dot{K} = \dot{Y} = n$. That is to say, capital, labour and output expand at the natural rate even in the short run. Of course they do so in the long run. This will be stated as $K = nK$.

Next catch a glimpse of public debt dynamics. The government provides a certain volume of goods and services per head $G = gN$ with $g = \text{const}$. In addition, the government borrows a fixed amount per head $B = bN$ with $b = \text{const}$. The budget deficit in turn augments public debt $\dot{D} = B$. Moreover the government collects a lumpsum tax $T = tN$ with $t = \text{const}$. The government budget identity is again $B + T = G + rD$. Taking account of the functional relationships, this can be put as $bN + tN = gN + rD$. Here the government presets purchases per head and the budget deficit per head, while it must adjust the lumpsum tax.
Beyond this we throw some light on the dynamics of foreign assets. Domestic residents earn the interest rate $r$ on foreign assets $F$, hence the interest inflow totals $rF$. Disposable income embraces factor income, public interest and the interest inflow, net after tax respectively $Y_d = Y + rD + rF - T$. Households save a constant fraction of disposable income $S = sY_d$. The current account surplus is identical to the excess of savings over the budget deficit and investment $E = S - B - I$. The current account surplus in turn adds to foreign assets $\dot{F} = E$. Backward substitution affords $\dot{F} = s(Y + rD + rF - T) - B - I$. Paying heed to $B + T = G + rD$, $B = bN$, $G = gN$, $\dot{D} = B$ and $nK = K = I$, this can be written as $\dot{F} = s(Y + rF + bN - gN) - \dot{D} - nK$.

After these introductory remarks, the short-run equilibrium can be described by a system of five equations:

$$Y = K^\alpha N^\beta$$  \hspace{1cm} (1)
$$r = \alpha Y / K$$  \hspace{1cm} (2)
$$\dot{D} + \dot{F} = s(Y + rF + bN - gN) - nK$$  \hspace{1cm} (3)
$$\dot{D} = bN$$  \hspace{1cm} (4)
$$\dot{N} = nN$$  \hspace{1cm} (5)

Here $\alpha$, $\beta$, $b$, $g$, $n$, $r$, $s$, $D$, $F$ and $N$ are exogenous, whereas $\dot{D}, \dot{F}, K, \dot{N}$ and $Y$ are endogenous.

Last but not least, it is required to do the analysis in per capita terms. Take the time derivative of foreign assets per head $f = F/N$ to realize $\dot{f} = \dot{F}/N - (F/N)(\dot{N}/N)$. Along these lines, the short-run equilibrium can be represented as follows:

$$y = k^\alpha$$  \hspace{1cm} (6)
$$r = \alpha y / k$$  \hspace{1cm} (7)
$$\dot{d} + \dot{f} = s(y + rf + b - g) - nd - nf - nk$$  \hspace{1cm} (8)
$$\dot{d} = b - nd$$  \hspace{1cm} (9)

In this version $\dot{d}, \dot{f}, k$ and $y$ adapt themselves.