Abstract: In the summer of 1986 we carried out a large-scale experimental investigation of consumption under uncertainty. Our purpose was to test the predictions of the famous Hall theory of optimal consumption under income uncertainty, and, in particular, to test the proposition (revealed to us in a preliminary pilot study) that behaviour is not optimal in an absolute sense, but that its comparative static implications agree with those of optimality theory. In total, 128 subjects performed the experiment: 8 for each of 16 parameter sets. Preliminary analysis lends support to our proposition.

1. Introduction

This paper reports on one of a series of large-scale experimental investigations into economic behaviour under conditions of uncertainty carried out at EXEC, the Centre for Experimental Economics at the University of York. These investigations, and, indeed the Centre itself, were motivated by the belief that "optimality economics" does not provide all the answers to economic behaviour, combined with the belief that experimental methods can yield new insights into such behaviour.

This paper contains a preliminary analysis of a large-scale experimental investigation into "consumption under uncertainty". Optimality economics has a well-developed body of literature - both theoretical and empirical - on this topic, and our desire in this set of experiments was to see how closely the predictions of the optimality theory fit the actual behaviour of subjects in an experimental environment. In addition, we were concerned to try to explain why the optimality theory appears to perform pretty well when tested empirically at a fairly aggregate level using cross-section and time-series data, while the foundation of the theory (that is, the axiomatic base to that theory) appears to be rather lacking in realism.

First, we outline the optimality theory. Then we describe our experimental design. Then we examine briefly a pilot study we carried out with 14 subjects. Finally, we describe and analyse the large-scale study itself.

2. The economic theory of optimal consumption under uncertainty

As our desire was to test optimality theory with the closest confrontation possible, we deliberately chose the simplest stylised model that we could find. This
is an adaptation (simplification) of a very famous model immortalised in an equally famous paper by Hall (1978).

We consider a discrete (random horizon) world in which we envisage an economic agent with a well-defined objective function (which we shall specify shortly) faced with a random income stream and a decision-problem involving the optimal allocation of this income stream on a consumption stream. We assume, for the time being, that the individual can freely borrow and lend at a known rate of interest.

Let us be more specific. We consider a discrete random-horizon model, in which the (uncertain) income stream is denoted by \(Y_1, Y_2, \ldots, Y_t, \ldots\) and the consumption stream by \(C_1, C_2, \ldots, C_t, \ldots\). We denote the individual's wealth at the beginning of period \(t\) by \(W_t\), and we let \(r_t\) denote the (certain) rate of return on wealth held at the end of period \(t\) (\(r_t\) equals 1 plus the rate of interest). We assume, for simplicity, that the individual's objective, as viewed from the beginning of period \(t\), is to maximize

\[
E_t \sum_{T=t}^{\infty} \rho^{T-t} U(C_T)
\]  

where \(E_t\) is the expectation as viewed from the beginning of period \(t\), and where \(U(.)\) is the individual's (per-period) utility function. The parameter \(\rho\) is the (assumed constant) product of the individual's discount factor and the probability that the individual will be alive next period given that he or she is alive this period.

We must now make some assumptions about timings: we assume that income in \(t, Y_t\), is known before the decision on \(C_t\), consumption in \(t\), is made. Furthermore, let us assume that \(W_t\) includes \(Y_t\). Finally, let us assume that \(C_t\) is chosen at the beginning of \(t\), immediately after \(Y_t\) is revealed. We thus have

\[
W_{t+1} = r_t(\begin{array}{c}W_t - C_t \end{array}) + Y_{t+1}. \tag{2}
\]

We now determine the individual's optimal consumption strategy. To this end, we introduce \(V_t(W_t)\) to denote the maximal value of (1), that is, of expected lifetime utility (as viewed from the beginning of period \(t\)). Obviously this depends on \(W_t\), the individual's wealth at the beginning of period \(t\). (It also depends on the various parameters of the problem, plus the joint distribution of \(Y_{t+1}, Y_{t+2}, \ldots\).) Thus, definitionally, we have

\[
V_t(W_t) = \max_{C_t, W_{t+1}, \ldots} \left\{ E_t \sum_{T=t}^{\infty} \rho^{T-t} U(C_T) \right\}. \tag{3}
\]

From this, it follows that

\[
V_t(W_t) = \max_{C_t} \left\{ U(C_t) + \rho E_t V_{t+1}(W_{t+1}) \right\}, \tag{4}
\]

where \(W_{t+1}\) is given by (2).