Chapter 3

The Poisson Forest

In the following pages and, except when otherwise specified, until the end of this work, we shall concentrate our attention on the case of a unique object or phenomenon -- a forest, a mineral deposit, a mountain range etc... -- which occupies a well-defined bounded part of that space in which we live. These objects are interesting in themselves. They can be, and actually are, studied from the panscopic viewpoint which is that of scientific enquiry. But they are also valuable and useful. The practitioner who is responsible for administering and exploiting them -- the forester, the miner -- meets problems of an essentially practical nature that he must solve, in one way or another, in order to fulfil his duties: for example, the miner must estimate the various parts of the deposit before he can decide which ones to mine and which ones to abandon because the grade is too low. In order to carry out this estimation, he has only at his disposal, by the very nature of things, very fragmentary and limited information. His viewpoint is therefore essentially monoscopic. Not that he has no interest in the other aspects of the object: on the contrary, one observes among professionals a sort of contained enthusiasm. But he must first of all think of fulfilling his task. In what follows, I shall sometimes adopt one viewpoint and sometimes the other, but the balance will be mostly in favor of monoscopic models. For it is in relation to the latter that the problem of objectivity arises in its most acute form. They are also the ones about which I have acquired the more direct experience through extensive practice. I can therefore hope that I shall know more or less what I am talking about when I speak of them.

External objectivity, that is, the objectivity of the general methodology we use to construct individual models, does not raise major problems: it is simply the sanction of practice. For example, Geostatistics can be considered as being objectively based because it has been successfully used to estimate several hundred mineral deposits. We shall therefore mainly deal, in what follows, with internal objectivity: that of statements concerning this particular deposit or that particular forest. We shall start with an introductory example, that of the “Poisson forest”.

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The Parameter $\theta$: Does It Exist and is It Useful?

In order to get to the heart of the matter in a concrete way let us consider the Poisson model sometimes used by foresters to represent the distribution of trees (considered as points) over the forest (J. P. Marbeau 1). Clearly, to say that the forest "is" Poisson is a condensed way of saying that the available data are not incompatible with this Poisson interpretation. According to the viewpoint of "orthodox" statistics, the most important problem one must then solve is that of "statistical inference", that is, the "estimation" of the unknown intensity $\theta$ of the Poisson process. For once this parameter is known one can calculate all other characteristics of the process. This point of view attributes, implicitly, a real and objective existence to the intensity parameter $\theta$: namely that even if our information allows us to arrive at no more than an approximate estimate of the "true" value of $\theta$, this does not change the fact that the latter exists somewhere in nature, and could be precisely measured, given perfect information. But in reality it is not at all certain that this contention has an operational sense: for in order to determine $\theta$ precisely, the forest would have to extend to infinity (and remain Poisson) while its real size is in fact limited. The presumed obviousness of the existence of $\theta$ is based on a summary identification of the model (the Poisson process) with reality (the forest). Such a confusion, which is quite common among statisticians, is essentially an epistemological short-circuit. For however well a model is adapted to its object, we never have a guarantee that all its characteristics will faithfully mirror objective properties of reality which can be put in a one-to-one-correspondence with them.

If we firmly maintain this necessary distinction between the model and reality, the problem of statistical inference, (which is from the rigorous point of view insoluble) loses much of its importance. The parameter $\theta$ remains partly indeterminate, even if we have perfect knowledge of the forest. One may therefore seriously doubt its "objective existence", and therefore not take too seriously its indeterminacy. In fact, the practical problem which interests the forester is never the estimation of the Poisson intensity $\theta$ of the theoretical model. The latter, even when used, only intervenes as a convenient computational intermediary and always disappears from the final result. What the forester really wishes is, for example, to estimate the number $N(S)$ of trees in a given area $S$, knowing the number $N(s)$ of trees counted in the sampled area $s$. Or else (and this is the same thing) he will be interested in the mean number $\langle S \rangle = N(S)/S$ of trees per hectare in the area $S$. To estimate $\theta(S)$ and $N(S)$ he will calculate the quantities:

$$\theta^*(S) = N(s)/s, \quad N^*(S) = SN(s)/s$$

(1)

It is to be noted that the Poisson interpretation actually plays almost no part in the formation of these "estimators", which appear to be the most "natural" possible. In fact, these estimators turn out to be unbiased in the framework of a much less restrictive interpretation than the Poisson model. It is enough, for example, to postulate a certain type of stationarity (in the probabilistic sense of