IV. DYNAMIC MODELS OF RESOURCE EXTRACTION AND DUOPOLISTIC MARKETS

This chapter introduces some models of oligopolistic markets for finite, nonrenewable depletable natural resources. Only oligopolistic models with a game theoretical approach are analyzed. A distinction is made between models with discrete time parameter and with continuous time parameter on the one hand, and between closed-loop controls and open-loop controls on the other hand. In case of closed-loop controls each oligopolist or player can redecide on his future strategy in considering the state of the game at each instant of time. In case of open-loop control the players can "only have a look at the watch", i.e. they cannot reconsider the state of the game and consequently they have to fix their strategies at initial time for the whole period under consideration.

In case of quantity strategies the amount \( x(t) \) of the resource, still available at time \( t \), is the state variable and the extraction rate \( q(t) \) the control variable. In case of price strategies the prices, fixed by the oligopolists, are the control variables, and the state \( x(t) \) is determined by the extraction rates \( q_i(t) \) corresponding to the prices \( p_i(t) \) as given by the demand function.

Assuming the extractable resource to be a homogeneous good, in case of price strategies the initial endowments of the resource play an important role, hence the discussion of asymmetric oligopolies, i.e. nonsymmetrically endowed oligopolies, is of interest. The solution concept chosen is the non-cooperative Nash equilibrium. Although this concept implies some shortcomings (changes of the strategy of one player are compared to each others isolated from joint changes of the strategies of all other players) it makes economically sense. Once reached, a Nash point is stable, since sliding away from it, a player has to expect a worse result. Therefore it is a sensible setup in our context, since we assume that all suppliers compete with one another, or, if they form a cartel, this cartel competes with those suppliers, who do not participate at the cartel. Although Stackelberg solutions in particular for asymmetric oligopolies could be of interest, they are not analyzed. Stackelberg equilibria are dynamically inconsistent in the sense that they depend on
the initial point of time: Suppose, a Stackelberg model is treated with initial time 0 and thereafter once again with initial time $t_0 > 0$ with values, given by the first solution at time $t_0$, then both solutions can differ from one another after time $t_0$ (see e.g. Simaan and Cruz, 1973b).

IV.1. An Introductory Duopoly Model with Production Strategies

For illustration we analyze a special case of a duopoly model of Burgermeister (see Krelle, 1976, pp.457). In accordance with our assumptions the commodity supplied, cannot be steadily produced, yet is available at a finite amount.

Each duopolist $i$ ($i=1,2$) can control his extraction rates $q_i(t)$, which he supplies. His initial endowments $x_i^0$ with the resource are given. Further there are restrictions related to the change of the control variables:

$$
\sigma_i^--\frac{dx_i(t)}{dt}<\sigma_i^+, \ t \in [0,T].
$$

The costs for the duopolists are given by

$$
c_i(t) = c_i(x_i(t)) = c_i(q_i(t));
$$

i.e. each duopolist owns an initial endowment $x_i^0$ of the resource, which can be changed by extraction; in this case it can only be diminished. The smallest and greatest change are given by the resource restriction:

$$
\sigma_i^- = 0 \text{ and } \sigma_i^+ = x_i(t), \ (t \in [0,T]; \ i=1,2).
$$

The costs are assumed to be independent on the remaining amount of the resource stock and are related to the extraction rate $q_i(t)$.

Example 1 (linear demand, linear costs):

For a special case of duopoly the model can be treated analytically as follows: