Buckling Stresses of Shells Having Negative Gaussian Curvature

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Summary
In addition to geometric parameters buckling of shells of negative Gaussian curvature depends mainly on the membrane stress state present. According to buckling tests the effect of boundary conditions is relatively low whereas the numerical results are highly dependent on the boundary conditions. Again according to tests the hyperboloidal shell has a higher buckling resistance than a cylindrical shell having the same throat radius. The buckling resistance of hyperboloidal shells against circumferential compression can be increased by arranging stiffening rings of adequate size and number. Due to nonlinear behavior of reinforced concrete the buckling load factor drops to the half or less of the value obtained assuming a linear elastic behavior. This high reduction is mainly due to orthotropy and descending tangent modulus at the near of ultimate load of reinforced concrete cooling tower shells.

Introduction
Hyperbolic natural-draught cooling towers are already the largest reinforced concrete shell structures being built. Today there is a tendency to ever increasing height and diameter of the cooling towers keeping the wall thickness of the shell as small as possible for economy. For this reason stability behavior is a vital concern in design of the cooling tower shell having negative Gaussian curvature.

Although the buckling of cylindrical and spherical shells has been investigated from the beginning of this century the history of the buckling investigations on shells of negative Gaussian curvature is only 15 years young. One of the early researchs is done by Stein and McElman[1] who calculated the buckling load of bowed-in and bowed-out toroidal segments.
under lateral or hydrostatic external pressure. On the basis of classical buckling analysis the buckling load of the bowed-out shells was several orders of magnitude larger than that of the bowed-in configuration. Hutchinson[2] studied the imperfection sensitivity of the same shells within the framework of Koiter's general theory of initial post-buckling behavior [3]. It turned out that the bowed-in shell has a lower imperfection-sensitivity associated with its considerably lower classical buckling load.

The catastrophic collapse of some of the cooling towers in Ferrybridge/England in November 1965 gave impulse to the intensive research on the buckling of hyperbolic cooling tower shells at the end of the sixties. Till today the most extensive tests on cooling tower shaped models in wind tunnel have been carried out by Der and Fidler [4]. Using both electroformed copper and molded PVC models Der and Fidler measured the dynamic head required to buckle their models and derived from the test results Formula (1) for the buckling pressure of a hyperboloid with proportions similar to the models

\[ q_{cr} = c \cdot E \left( \frac{t}{R_T} \right)^n \]

In Formula (1) E is the modulus of elasticity, t wall thickness of the shell and \( R_T \) throat radius of the hyperboloid. Der and Fidler give for n the value 2.3 and for c an average value of 0.064. Ewing [5] treated the stability problem of cooling tower shells analytically for axisymmetric pressure and non-axisymmetric wind load and found the value of 7/3 for n.

Veronda and Weingarten [6] have tested a large number of hyperboloidal shell models made of PVC sheets by bonding along a longitudinal seam. Due to thermoforming the models were getting thinner with increasing distance from the throat. On the whole the models were very likely afflicted with initial imperfections in the shape due to relatively small wall thickness which was only 0.25 mm. The buckling began with a single dimple which mostly shapes near the upper boundary, where the models were