Rheometry of viscoplastic dispersions

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Abstract: Phenomenological models for viscoplastic materials are constructed. The response to an oscillatory shear experiment is derived and found to agree with experimental results.

Key words: Viscoplasticity, yield stress, oscillatory shear

1. Introduction

In the literature several models have been proposed to describe fluids which exhibit a yield stress [1] but most of them are not suitable for describing the dynamic behaviour of such viscoplastic fluids. This paper describes three groups of viscoplastic models. The dynamic properties of these models are calculated and compared with experiments. In the last section some possible extensions are indicated.

2. Phenomenological description of viscoplastic behavior

We have developed viscoplastic models by generalising known fluid models with a plastic structure. For the description of plastic behaviour we used the expression for the Von Mises plastic material [2]. In shear flow this reduces to

\[ \tau = \tau_0 \cdot \text{sgn}(\dot{\gamma}) \]  

(1)

The behaviour of this model is represented by a slider in a mechanical analogue. With this element we constructed three groups of viscoplastic models with non elastic-plastic, elastic-plastic and viscoelastic-plastic structures, as shown in figure 1. Since the plastic structure is in parallel with the fluid model, the response of the total model is found by adding the responses of both branches. We now consider the re-
The shear stress in the non elastic-plastic structure is given by

$$\tau_p = \tau_0 \cdot \text{sgn}(\cos \omega t) . \tag{2}$$

Fourier decomposition of eq. (2) yields

$$\tau_p = \frac{4 \tau_0}{\pi} \sum_{p} \frac{\cos(p \omega t)}{p}, \quad p = \text{odd} . \tag{3}$$

If we restrict attention to the transfer of the fundamental frequency, then $\eta'_p$ and $\eta''_p$, being the components of the complex viscosity of the plastic structure, will be

$$\eta'_p = \frac{4 \tau_0}{\pi \gamma_0 \omega}, \quad \eta''_p = 0 . \tag{4}$$

The total response, with respect to the fundamental frequency, thus becomes

$$\eta' = \eta'_j + \frac{4 \tau_0}{\pi \gamma_0 \omega} \eta'' = \eta''_j , \tag{5}$$

where $\eta'_j$ and $\eta''_j$ are the dynamic viscosities of the fluid model used. From this relatively simple model it can be seen that the measured response decreases with increasing strain amplitude. This is in accordance with other observations [3, 4, 5].

A somewhat more realistic model is obtained with an elastic-plastic structure. This model predicts a linear response if $\gamma_0 < \tau_0 / G$. In this case, the stress in the plastic structure is found to be

$$\tau_p = \tau_0 \quad \text{for} \quad \omega t < \pi / 2 , \tag{6a}$$

$$\tau_p = \tau_0 + G \gamma_0 (\sin \omega t - 1) \quad \text{for} \quad \pi / 2 < \omega t < \omega t_1 \tag{6b}$$

$$\tau_p = -\tau_0 \quad \text{for} \quad \omega t_1 < \omega t < 3 \pi / 2 \tag{6c}$$

$$\tau_p = -\tau_0 + G \gamma_0 (\sin \omega t + 1) \quad \text{for} \quad 3 \pi / 2 < \omega t < \omega t_2 \tag{6d}$$

with

$$\sin \omega t_1 = 1 - 2 \tau_0 / G \gamma_0 , \quad \omega t_2 = \omega t_1 + \pi . \tag{7}$$

The response of a Newtonian fluid generalised with an elastic-plastic structure is depicted in figure 3. The viscoelastic-plastic structure gives linear behaviour not only if $\gamma_0 < \tau_0 / G$ but also at frequencies below a critical frequency, $\omega_c$, given by

$$\omega_c = 1 / \sqrt{\left( \frac{G \gamma_0}{\tau_0} \right)^2 - 1} , \quad \lambda = \eta / G . \tag{9}$$

The occurrence of a critical shear rate or frequency is in accordance with observations reported in [5, 6]. This model gives:

$$\eta' = \eta'_j + \frac{\eta}{\pi(1 + \lambda^2 \omega^2)} \left[ \omega t_2 - \omega t_1 ight.$$

$$\left. - \frac{1}{2} \sin 2 \omega t_1 + \frac{1}{2} \sin 2 \omega t_2 + \lambda \omega (\cos^2 \omega t_2 - \cos^2 \omega t_1) \right] + \frac{2 \tau_0}{\pi \gamma_0 \omega (1 + \lambda^2 \omega^2)} \cdot \left( \sin \omega t_1 + \sin \omega t_2 + \lambda \omega (\cos \omega t_1 + \cos \omega t_2) \right) \tag{10}$$

$$\eta'' = \eta''_j + \frac{\eta}{\pi(1 + \lambda^2 \omega^2)} \left[ \sin^2 \omega t - \sin^2 \omega t_1 ight.$$

$$\left. + \lambda \omega (\omega t_2 - \omega t_1) + \lambda \omega (\cos \omega t_2 \cos \omega t_1 - \sin \omega t_1 \cos \omega t_2 - \cos \omega t_1 \cos \omega t_2) \right]$$

$$+ \frac{\lambda \omega}{1 + \lambda^2 \omega^2} (\sin \omega t_1 + \sin \omega t_2)$$

$$+ \lambda \omega (\cos \omega t_1 + \cos \omega t_2) - \cos \omega t_1 - \cos \omega t_2 \right) \tag{11}$$

Figure 2 shows the shear stress in the plastic structure for some values of $G \gamma_0 / \tau_0$. Again $\eta'$ and $\eta''$ are found after Fourier decomposition to be

$$\eta' = \eta'_j + \frac{G}{\pi \omega} \left( \frac{4 \tau_0}{G \gamma_0} \left( 1 - \frac{\tau_0}{G \gamma_0} \right) \right),$$

$$\eta'' = \eta''_j + \frac{G}{\pi \omega} \left[ \omega t_1 - \frac{1}{2} \pi + \frac{1}{2} \sin 2 \omega t_1 \right]. \tag{8}$$