Spectral Analysis of Non-Stationary Time Series

I. G. Zhurbenko, Moscow

The last decade has seen a great popularity of application of the methods of the correlation and spectral analyses of time series [1-9]. The spectral approach to researches had a necessary condition: a given model of the series must be stationary. But in practice in most researches this condition is possible only within some limited time interval or impossible at all. At the same time a statistical study of the spectra of these series is drawing more and more attention [4-12] despite the qualitative and technical complexity of models. Spectral estimates obtained by time shift (they were systematically studied in [8,9]) suggested a new approach which is simple and convenient in most situations. Below is the study of a time series model which not only substantiates investigation of spectra changing in time but also brings us to the problems of the multivariable statistical analysis of these spectra: pattern recognition in the spectral domain, spectral disorders, spectral investigation when there are many strong non-stationary phenomena, and others. The size of the paper and some methodical considerations prevent demonstration of application of this approach to various problems (its simplicity makes these problems quite easy and particularly logical from the researcher's viewpoint. At present on the basis of the enterprise "Spectrum" a group of
scientists (I.A. Kozhevnikova, I.G. Nideker and the author of this paper) are working on a pocket of programs RSA on the time series analysis including the above-mentiond approach. The pocket with illustrations is intended for a wide group of researchers who have a good intuition and educational background in the field of their researches but superficial knowledge of time series analysis.

The main model

Let \( X(t,u) \) be a time series which is stationary in a wide sense with respect to the discrete parameter \( t \) and dependent on the continuous real parameter \( u \). Besides, we shall consider the following conditions satisfied:

I. There exist uniformly bounded absolute moments up to the fourth order including for the entire set of random quantities \( X(t,u) \):

\[
E|X(t,u)|^k \leq E|X|^k, \quad k=1,2,3,4. \tag{1}
\]

With respect to the parameter \( u \), the random quantities \( X(t,u) \) are continuous in the mean square and

\[
E|X(t,u+\Delta u) - X(t,u)|^2 \leq \int_0^2 |\Delta u| E|X|^2 \tag{2}
\]

holds. The coefficient \( \int_0^2 \) indicates the speed of deviation from stationarity along the parameter \( u \). In some cases we shall require that the fourth cumulants \([1,9]\) of the random quantities \( X(t,u) \) should be continuous with respect to the parameter \( u \) as well:

\[
|S_4(X(t,u+\Delta u)) - S_4(X(t,u))| \leq \int_0^2 |\Delta u| E X^4 \tag{3}
\]