Subject: Sales Response to Advertising and Promotion

Title: Estimating Dynamic Effects of Market Communications Expenditures

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Summary: The problem is the measuring of market response to a "communications mix"—the various means which a firm employs to transmit sales messages to potential buyers. Distributed lag models are applied to time series data for an ethical drug to estimate the short-run, intermediate, and long-run effects on market share of expenditures made for journal advertising, direct mail advertising, and samples and literature. Important differences are found among the communications variables with respect to the magnitude and over-time pattern of effect each had on market share. The results indicate that the average historic allocation of resources to alternative communication vehicles has been in inverse relation to measured market response.

Model: The basic distributed lag model is:

\[
\text{LMS}(t) = a_0 + \sum_{i=0}^{I} a_{i+1} \text{LJA}(t-i) + \sum_{j=0}^{J} b_{j+1} \text{LSL}(t-j) + \sum_{k=0}^{K} c_{k+1} \text{DM}(t-k) + e(t),
\]

where

- \( L \) = the log of a variable
- \( MS \) = market share
- \( JA \) = journal advertising
- \( SL \) = samples and literature
- \( DM \) = direct mail advertising
- \( e(t) \) = residual.

Whenever a communication variable is zero for some period \( t \), it is set equal to $1 for that \( t \) to avoid a log value of minus infinity. In view of the problems of multicollinearity and degrees of freedom we modify the model:

\[
\text{LMS}(t) = a_0 + a_1 \text{LJA}(t) + a_{12} \text{LJA}(t-1) + a_{11} \text{LJA}(t-2) + \sum_{i=0}^{8} \lambda^i \text{LJA}(t-i) + b_1 \text{LSL}(t) + b_{11} \text{LSL}(t-1) + \sum_{i=0}^{3} \lambda^i \text{LSL}(t-i-1) + c_1 \text{DM}(t) + \sum_{i=0}^{2} \lambda^i \text{DM}(t-i) + e(t),
\]
where \( 0 < \lambda < 1 \).

The following points should be noted:

(a) The geometric decay in the effect of the communication variables may set in at different points in time for each variable.

(b) Once the geometric decay sets in, the same rate of decay \((1-\lambda)\) is assumed to hold for all exogenous variables.

(c) The decay terms form an infinite series.

(d) Specific lags are included because there is reason to believe that certain of the variables may have a greater effect after one or two periods than they do in the period during which the expenditure was made. In addition, the specific lags allow each variable to exhibit an individual decay rate up to the period in which the geometric decay sets in.

The model in (2) may be transformed into a form which is readily estimated. First, write (2) for \(LMS(t-1)\), multiply both sides through by \(\lambda\), and subtract it from (2). This yields:

\[
LMS(t) = a_0 (1-\lambda) + a_1 LJA(t) + (a_2 - \lambda a_1) LJA(t-1) \\
+ (a_3 - \lambda a_2) LJA(t-2) + (a_4 - \lambda a_3) LJA(t-3) \\
+ b_1 LSL(t) + (b_2 - \lambda b_1) LSL(t-1) + (b_3 - \lambda b_2) LSL(t-2) \\
+ c_1 LDM(t) + (c_2 - \lambda c_1) LDM(t-1) \\
+ \lambda LMS(t-1) + e(t) - \lambda e(t-1) \\
= a_0 + a_1 LJA(t) + a_2 LJA(t-1) + a_3 LJA(t-2) \\
+ b_1 LSL(t) + b_2 LSL(t-1) + b_3 LSL(t-2) \\
+ c_1 LDM(t) + c_2 LDM(t-1) + c_3 LDM(t-2) \\
+ \lambda LMS(t-1) + U(t),
\]

where \(U(t) = e(t) - \lambda e(t-1)\) and