Parametric Correction of Finite Element Models Using Modal Tests

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Summary
This paper concerns the computational aspects of a new correction method for the control and the refinement of a given finite element model using modal tests results, when finite element models represent complex structures. The updating strategy is iterative and each iteration is divided into two stages: first, the local error measure on the constitutive relation is used to locate the most erroneous areas, second, the mass and stiffness matrices are corrected. Some examples illustrate this strategy.

Introduction
A recent tendency in engineering is to reduce the number of tests and to prefer numerical simulations even for complex structures; the tests are used to validate and verify the model. Tuning problems appear for many free-vibration industrial problems where the results obtained with finite element models are not too far from the observed experimental results. Nevertheless, differences can be observed: they proceed from erroneous estimation of the parameters describing the mass and stiffness properties. These errors are frequent in the modelling of joining sub-structures. The strategy, established by [LAD 89] [REY 90], is based on the concept of the error on the constitutive relation, and on the hypothesis that the experimental eigenvalues have generally a satisfactory quality. We present particularly in this paper the computational aspects of the new parametric correction method that uses the global error on the constitutive relation for the whole structure to quantify both the mass and stiffness matrices.

Different methods have been proposed to solve tuning problems: Baruch [BAR 82], Berman [BER 71], Chen [CHE 89], Caesar [CAE 84], Ewins [EWI 88], Link [LIN 87] construct directly the corrected mass and stiffness matrix. Collins [COL 74] and Zhang [ZHA 87] apply a sensitivity analysis. Other approaches based on the residues computation of equilibrium equations have been described by Cottin [COT 84] and Berger [BER 89].

Our strategy is an iterative parametric strategy where each iteration needs two stages: first, the local error measure on the constitutive relation is used to locate the most erroneous areas, and second, the corrections of the involved structural parameters are computed. The iterative process
is stopped, when the global error computed for the whole structure and all the experimental modes is less than an accuracy \( \varepsilon_0 \) fixed by the user.

### The Error Measure on the Constitutive Relation

The data of the tuning problem are: the finite element model, the experimental eigenvalues \( \lambda_i \), i.e. \([1, q]\) for the q "measured" modes, the experimental eigenshapes \( \Pi U_i \), where \( \Pi \) is the projection operator indicating that the experimental eigenshape is partly measured.

The starting point is to assume that the quality of the experimental eigenvalues is good enough, i.e., the model has to give the experimental eigenvalue \( \lambda \).

Then, let \( \Omega \) be a bounded subset with the boundary \( \partial \Omega \) corresponding to the structure. To specify the boundary conditions, let us consider two complementary subsets \( \partial_1 \Omega \) and \( \partial_2 \Omega \). Let the displacement field be given on \( \partial_1 \Omega \) and the stress field be given on \( \partial_2 \Omega \). And let be \( U = \{ U', U' | \partial_1 \Omega = 0, \ U' \ \text{regular} \} \), we then search to construct the complete associated eigenshapes by solving for each given \( \lambda \) the following problem \( P_1^\lambda \):

To find a couple \( (U, \sigma) \) where \( U \) is a displacement field and \( \sigma \) a stress field, such as:
- \( U \) satisfies the kinematic constraints, \( U \in U \) (1)
- \( (U, \sigma) \) satisfies the equilibrium equation:

\[
\forall U^* \in U \int_{\Omega} \text{Tr}(\sigma \varepsilon(U^*))d\Omega = \lambda \int_{\Omega} \rho \ U^*d\Omega \quad (\rho \ \text{density}) \quad (2)
\]

- and such as \( (U, \sigma) \) verifies the constitutive relation \( \sigma = K \varepsilon(U) \).

\( K \) is Hooke's operator and \( \varepsilon \) the strain operator, (1) and (2) mean \( (U, \sigma) \) is an admissible couple. Let us introduce: \( A_d = \{ (U', \sigma') , \ U' \ \text{regular}, \ U'|_{\partial_1 \Omega} = 0, \ \text{and} \ (U', \sigma') \) verifying \( \forall U^* \in U \int_{\Omega} \text{Tr}(\sigma' \varepsilon(U^*))d\Omega = \lambda \int_{\Omega} \rho \ U' U^*d\Omega \quad (\rho \ \text{density}) \} \).

An equivalent problem is to search, for the given experimental eigenvalue \( \lambda \), the admissible couple \( (U, \sigma) \) where \( U \) is a displacement field and \( \sigma \) a stress field, such as they minimise on \( A_d \) the error measure on the constitutive relation:

\[
J : (U', \sigma') \rightarrow J (U', \sigma') = \| \sigma' - K \varepsilon(U') \|^2 \quad \text{with} \quad \| \sigma' \| = \int_{\Omega} \text{Tr} (\sigma' K^{-1} \sigma') d\Omega
\]

We obtain a displacement approach associating to the stress field \( \sigma' \) the displacement field \( V' \in U \) solution of the following elastic problem:

\[
\forall U^* \in U \int_{\Omega} \text{Tr}(K \varepsilon(V') \sigma')d\Omega = 0
\]