This chapter introduces shape descriptors and uniform coverings of shapes found in digital images. Shape descriptors are useful in representing and extracting shape information from images. In general, a digital image shape descriptor is an expression that describes, identifies or indexes an image region. Shape descriptors are usually mathematical expressions used to extract image region shape feature values. The basic approach is to use shape descriptors to represent, extract and quantify information about image shapes.

The study of shapes in digital images leads to shape set patterns. Two basic types of shape set patterns are commonly found in the study of image
patterns, namely, spatial shape set patterns and descriptive shape set patterns (see, e.g., [249, 95, 93, 94, 114, 194, 218, 215]). A spatial shape set pattern is a collection of shape point sets that have nonempty intersection.

**Example 10.1. Spatial Penrose Tiling Shape Pattern.**
Tile $t_1$ in the penrose tiling in Fig. [10.3] is spatially near tiles $t_2, t_3, a$ as well as a number of other unlabelled tiles that are touching some part of $t_1$, i.e., $\text{cl} t_1 \cap \text{cl} t_2 \neq \emptyset, \text{cl} t_1 \cap \text{cl} a \neq \emptyset$, and so on. These observations lead to the spatial shape pattern

$$\mathcal{P}(t_1) = \{t_1, t_2, t_3, a, \ldots\}.$$  

A descriptive shape set pattern is a collection of shape point sets that have nonempty descriptive intersection with the pattern motif, i.e., given shape point sets $M, A$ in $\mathcal{P}_\Phi(M)$, then $M \cap A \neq \emptyset$. In this form of shape pattern, patterns are found after choosing a set of probe functions $\Phi$ representing shape features and, possibly, other features such as colour or intensity and choosing one or more pattern generators.

**Example 10.2. Descriptive Penrose Tiling Shape Pattern.**
Choose $\Phi$ to be a set of probe functions representing shape features such as connected, edge gradient, and edge gradient orientation as well as colour and intensity features. Also, for example, choose tile $t_1$ in Fig. [10.3] as a shape pattern generator. Tile $t_1$ in the penrose tiling in Fig. [10.3] is descriptively near tiles $m_1$ (in the middle tiling) and $r_1$ (in the righthand tiling) as well as a number of other unlabelled tiles that are descriptively near some part of $t_1$. In generating descriptive shape patterns, we use the descriptive closure of a set $A$ in a picture $X$ (denoted by $cl_\Phi A$), defined by

$$cl_\Phi A = \left\{ x \in X : x \cap A, i.e., \Phi(x) \in \mathcal{Q}(A) \right\}.$$  

Then

$$\mathcal{P}_\Phi(t_1) = \{t_1, m_1, r_1, \ldots\}.$$  

That is, $cl_\Phi t_1 \cap cl_\Phi m_1 \neq \emptyset$, since the gradient orientation of edges along the border of $t_1$ matches the gradient orientation of the edges along the border of $m_1$. Similarly, $cl_\Phi t_1 \cap cl_\Phi r_1 \neq \emptyset$, and so on.