This chapter focuses on the detection of edges, lines, ridges and corners in digital images. Interest in ridges in digital images began in the early 1980s with the study by R. Haralick on zero crossing second derivative edge detection [97] and the subsequent work by T. Lindeberg in the late 1990s [157]. In nature, a ridge is a long, narrow hilltop, mountain range or watershed. Descriptive uniform topologies and new forms of visual patterns are natural byproducts of the study of edges, lines and ridges.

In a digital image, a ridge is an edge that is concave down and, traveling up the concavity, the second derivative is negative for a function used to approximate a target ridge pixel. From a topological point of view, a ridge has interest because it can be viewed as set of boundary points in a closed set. In the search for near ridges, it is helpful to consider the size of the interior of a ridge and the distance between neighbouring corners such as the corners in a written signature or in handwriting samples. There are similar issues for corners in an image, since a corner like the one shown in Fig. 5.1 can be viewed as a closed set, if we cap off the base below a corner. Assuming that a ridge is a closed set, there is the interesting issue about the closeness or remoteness of nearby ridges (in an image such as a satellite image showing

![Sample edges, lines and ridges](image1)

**Fig. 5.1.** Sample edges, lines and ridges

![Ridge A Near Collar of Ridge E](image2)

**Fig. 5.2.** Ridge A Near Collar of Ridge E
Fig. 5.3. \( S_\varepsilon(E) \subset B \): \( B \) is a Proximal Neighbourhood of \( S_\varepsilon(E) \)

a landscape, there can be many ridges). Measurement of the closeness or remoteness of ridges in an image provides a means of classifying an image.

The nearness or remoteness of ridges can also be viewed in the context of collar sets, i.e., a ridge with an \( \varepsilon \)-wide border around the outside of a ridge. Then, for example, ridge \( A \) is near ridge \( E \) in Fig. 5.2 since the tip of \( A \) has points in the collar of \( E \). Ridges \( A \) and \( E \) in Fig. 5.4 are examples of collar-based near sets. With the introduction of ridges that are in the interior of collar sets, it is then possible to view the nearness of ridges in terms of whether or not points in one ridge belong to the collar surrounding another ridge. Let \( (X,d) \) be a metric space with subset \( A \) in \( X \) and let \( \varepsilon > 0 \). The \( \varepsilon \)-collar around \( A \) (denoted by \( S_\varepsilon(A) \)) is defined by

\[
S_\varepsilon(A) = \{ x \in X : d(x,A) < \varepsilon \}.
\]

A subset \( B \) in \( X \) is a proximal neighbourhood of \( A \), if and only if, \( B \) contains an \( \varepsilon \)-collar around \( A \) [56, §2.2].

Example 5.1. Notice that \( a \in S_\varepsilon(A) \) for all \( a \in A \), since \( d(a,A) < \varepsilon \). Hence, \( S_\varepsilon(A) \) is a proximal neighbourhood of \( A \). Another example of proximal neighbourhood is shown in Fig. 5.3 where \( B \) is a proximal neighbourhood of \( S_\varepsilon(E) \).

Thought Problem 34. Collar set.
Let \( X \) be a digital image and let \( E \) be a be any subset of \( X \). Choose \( \varepsilon > 0 \). Give an example of \( S_\varepsilon(E) \), the collar set of \( E \).

Thought Problem 35. Proximal neighbourhood.
Give an example of a proximal neighbourhood in a digital image.

Taking this a step further, one can compare the descriptions of points in a ridge set with the descriptions of points in a collar surrounding another ridge. This also means that the nearness of ridges can be determined without resorting to distance measurements, leading to a significant reduction in computational overhead.