This chapter introduces a number of selected topics that serve to strengthen the overall view of a topology of digital images. There is interest in identifying those parts of each image with uniform as well as interesting characteristics. This interest leads us to consider an introduction to what are known as principal axes in a division of image spaces into four quadrants (image niches) containing sets of pixel clusters and with the origin located at the centre of mass of an image.

The **principal axes** of a point sample are found by choosing the origin of the axes to be the centre of mass (centre of gravity). Principal axes are perpendicular to each other and along the principal axes the point sample has little or no spread (**minima of variance**) \[31\]. The unit vectors (vectors with a magnitude of 1) along the principal axes are called eigenvectors. In this work,
the point sample is a set of pixel intensities in a digital image. Principle axes provide a basis for what is known as principal component analysis. For PCA considered in the context of image processing, see [198]. Sample principal axes in a subimage and in a full image for a Manitoba honeybee are shown in Fig. 9.1. Again, for example, the arrows in Fig. 9.2 correspond to the principal axes oriented relative to point masses in an inertial system. A consideration of principle axes in digital images provides a natural transition to two forms of uniform structures (uniformities), namely, entourage uniformity [36, 120, 304] and covering uniformity [120 §1.1], [129 §6].

9.1 Principal Component Analysis

This section briefly introduces principal component analysis (PCA), which is a form of linear integral transformation that simplifies a multidimensional data set to a lower dimension. Here is a quick summary of PCA, inspired by J. Schlens [260, p. 9].

(PCA.1) Organize the data (in our case, pixels in an image) as an \( m \times n \), where \( m \) is the number of measurement types (in our case, each row of an image is considered a measurement type) and \( n \) is the number of samples (in our case, number of column vectors in an image). Let \( g \) denote an \( m \times n \) digital image.

(PCA.2) Subtract the mean for each measure type.

(PCA.3) Find the \([x, y]\), the row \( x \) and column \( y \) indices for each pixel.

(PCA.4) Find the mean value of \([x, y]\), using \( \text{mean}([x \ y]) \).

(PCA.5) Find the centroid

(PCA.6) Calculate the covariance matrix \( C \) from \([x, y]\).

(PCA.7) Calculate the eigenvectors \( V \) and eigenvalues \( D \) of the covariance.

(PCA.8) Use eigenvectors \( V \) and \( \text{mean}([xy]) \) to determine the principal axes.

Here are a number key terms to take into account in the study of PCA.

(term.1) covariance. Let \( A, B \) be a pair of \( m \times n \) matrices, where each row is an observation and each column is a variable. Further, let \( \bar{a}, \bar{b} \) be the mean values of \( A, B \), respectively. Then

\[
\text{covariance of } A, B = E[(A - \mu_A)(B - \mu_B)] = \frac{1}{n} \sum_{i=1}^{n} (a_i - \bar{a})(b_i - \bar{b}).
\]

For example, in Matlab

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