Chapter 6

Multi-criteria Fuzzy Portfolio Optimization

Abstract. In this chapter, we describe fuzzy portfolio selection models using five criteria: short term return, long term return, dividend, risk and liquidity. For portfolio return, we consider short term return (average performance of the asset during a 12-month period), long term return (average performance of the asset during a 36-month period) and annual dividend. This is done in order to capture subjective preferences of the investors for portfolio return. For a given expected return, the negative semi-absolute deviation is penalized which quantifies portfolio risk. Further, we categorize all individual investor attitudes towards bearing portfolio risk into one of the following two distinct classes: aggressive (weak risk aversion attitude) and conservative (strong risk aversion attitude). The nonlinear S-shape membership functions are employed to express vague aspiration levels of the investor regarding the multiple criteria used for portfolio selection.

6.1 Multi-criteria Portfolio Selection Model

Most of the existing portfolio selection models consider return and risk as the two fundamental factors that govern investors’ choice. However, it is often found that not all the relevant information for portfolio selection can be captured in terms of return and risk only. The other considerations/criteria might be of equal, if not greater, importance to investors. By considering these in the portfolio selection model, it may be possible to obtain portfolios in which a deficit on account of the return and risk criteria is more than compensated by portfolio performance on other criteria, resulting in greater overall satisfaction for investors. Thus, multiple criteria portfolio selection models have received great attention in recent past. Some of the relevant references in this direction include Arenas et al. [3], Ehrgott et al. [30], Fang et al. [33], Gupta et al. [41, 42, 43, 44], Li et al. [82], Nanda et al. [96].

Here, we formulate portfolio selection problem as an optimization problem with multiple objectives assuming that the investor allocate his/her wealth
among \( n \) assets that offer random rates of return. We introduce some notation as follows:

### 6.1.1 Notation

- \( r_i \): the expected rate of return of the \( i \)-th asset,
- \( x_i \): the proportion of the total funds invested in the \( i \)-th asset,
- \( y_i \): a binary variable indicating whether the \( i \)-th asset is contained in the portfolio, where
  \[
  y_i = \begin{cases} 
  1, & \text{if } i \text{-th asset is contained in the portfolio}, \\
  0, & \text{otherwise}, 
  \end{cases}
  \]
- \( d_i \): the annual dividend of the \( i \)-th asset,
- \( r_{i}^{12} \): the average performance of the \( i \)-th asset during a 12-month period,
- \( r_{i}^{36} \): the average performance of the \( i \)-th asset during a 36-month period,
- \( r_{it} \): the historical return of the \( i \)-th asset over the past period \( t \),
- \( u_i \): the maximal fraction of the capital allocated to the \( i \)-th asset,
- \( l_i \): the minimal fraction of the capital allocated to the \( i \)-th asset,
- \( L \): the minimum desired level of portfolio liquidity,
- \( \tilde{L}_i \): the fuzzy turnover rate of the \( i \)-th asset,
- \( h \): the number of assets held in the portfolio,
- \( T \): the total time span.

We consider the following objective functions and constraints in the multiobjective portfolio selection problem.

### 6.1.2 Objective Functions

#### Short Term Return

The short term return of the portfolio is expressed as

\[
f_1(x) = \sum_{i=1}^{n} r_{i}^{12} x_i,
\]

where \( r_{i}^{12} = \frac{1}{12} \sum_{t=1}^{12} r_{it}, \) for \( i = 1, 2, \ldots, n; \) \( r_{it} \) is determined from the historical data.