Multilevel Methods for Inverse Bioelectric Field Problems

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Abstract. The reconstruction of bioelectric fields from non-invasive measurements can be used as a powerful new diagnostic tool in cardiology and neurology. Mathematically, the reconstruction of a bioelectric field can be modeled as an inverse problem for a potential equation. This problem is ill-posed and requires special treatment, in particular either regularization or an otherwise suitable restriction of the solution space.

The differential equation itself can be discretized by finite differences or finite elements and thus gives rise to a large sparse linear systems for which multigrid is one of the most efficient solvers, but regularization, adaptive mesh refinement, and efficient solution techniques must be combined to solve the inverse bioelectric field problem efficiently. While multigrid algorithms can reduce the compute times substantially, new local regularization techniques can be used to improve the quality of the reconstruction. Local mesh refinement can be used to increase the resolution in domains of increased activity, but must be used with care because refined meshes worsen the ill-conditioning of the inverse problem.

1 Introduction

Hundreds of times each second, the brain sends electrical impulses racing through the body’s web of nerve cells to the motor neurons, where they initiate the chemical reactions that cause muscles to contract. Several decades ago, scientists recognized that these excitation currents produce an electrical field that can be detected as small voltages in the skin or scalp. By measuring changes in the pattern of the body’s electrical activity, they could detect some forms of heart diseases and neurological disorders. Electrocardiograms (ECGs), for the heart, or electroencephalograms (EEGs), for the brain, measure these voltages. However, because they are relatively coarse and imprecise techniques, they provide physicians with mere snapshots of heart or brain activity. These glimpses help doctors spot disorders but are often insufficient for accurate diagnosis. For the latter, doctors turn to other techniques — even, in rare cases, to exploratory surgery. For example, to determine whether a patient who is not responding to medication has an operable form of the disorder known as temporal lobe epilepsy, neurosurgeons open the cranium and attach electrodes directly to the brain to identify whether the abnormal...
Electrical activity is highly localized (thus operable) or diffused over the entire brain. Similarly, in clinical cardiology there are forms of abnormal heart rhythm (arrhythmias) which do not respond to drug treatment or to artificial stimulation and must be treated surgically. Determining just where to operate, however, is seldom possible with standard ECGs. Instead, such a determination requires either further investigation via a catheter or an open chest examination with a roving, hand-held electrode or a flexible grid of surface electrodes. The localization process is often lengthy and may be unsuccessful [10].

Electrocardiographic imaging, uses an inverse solution applied to electric potentials recorded on the body surface. The goal of the computation is then to reconstruct the complete distribution of electric potentials throughout the torso and the cardiac current source characteristics that produce those distributions. This gives a more complete and informative description of the source of abnormal cardiac activity. To produce such images it is necessary to have accurate individual geometric patient models, to compute the solution of field equations for the potential distributions. Regularization is needed to deal with the extremely ill-posed nature of the problem.

The detection of focal electrical activity in the brain can be characterized as an inverse source localization problem. Given a subset of electrostatic potentials measured on the scalp and the geometry and electrical conductivity properties within the head, the inverse problem is to calculate the current vectors and potential fields within the brain. Mathematically this can be stated as inversely solving Poisson’s equation for the internal current sources. Clinical applications of inverse source localization and imaging from the EEG are already part of practice in advanced neurological and radiological departments, but there remain significant limitations to the technique. Present estimates of the best-case accuracy of a localized dipole in the brain lie in the range of 7–10 mm, a relatively coarse error given the size scale of the brain.

In this paper we will describe the models used in bioelectric field computations and will report on research applying multilevel methods to solve large-scale ill-posed bioelectric field problems using realistic geometries. In particular, multigrid methods can be used as very efficient solvers for the associated forward problem and we will present our experience with algebraic multigrid methods (AMG) which are well suited to deal with complex geometries, as are characteristic of this application. In Sect. 5 we will extend standard regularization to become a local regularization technique and in Sect. 4 we will discuss adaptive mesh refinement methods and their use in inverse problems. In Sect. 7 we will present our numerical results.

2 Mathematical Models

In this article we discuss two types of inverse problems. The first inverse problem we consider is electrocardiographic imaging, which can be formulated