7. Impact of Inbound Flows

As revealed in the previous chapter, one of the major distinguishing characteristics of inventory control in a Reverse Logistics context is the need for integrating a largely exogenously determined goods inflow. In this chapter we address this issue in more detail and quantify the impact of inbound goods flows on inventory dynamics and appropriate control strategies. For the sake of focus we start from a basic situation allowing for a detailed analysis. Subsequently, potential extensions and limitations of this approach are discussed. As with many of the models discussed in the previous chapter, we therefore aggregate the two inventory types distinguished in the framework of Figure 6.1 into a single stock point. Moreover, we assume demand and returns to be independent. Finally, we do not include a disposal option, hence procurement is the only means to control the system. One may view this setup as the common core of the models discussed in Chapter 6. Our goal is to identify appropriate decision rules in the above setting and to investigate the impact of the return flow on the system’s performance.

The material of this chapter is organised as follows. In Section 7.1 we formally introduce our model and notation. Section 7.2 addresses the special case of unit demand and return quantities. Deriving analytic expressions for the relevant distributions we prove optimality of a conventional \((s, Q)\)-policy for the procurement decisions. What is more, we show the cost function to have the same structure as in a traditional inventory model without returns, which allows for an easy computation of optimal control parameter values. In the subsequent sections we extend this approach to the case of general demand and return distributions. Section 7.3 provides the key result, showing that our model can be transformed into an equivalent classical inventory model without returns if procurement is controlled by an \((s, S)\)-order policy. Optimality of an \((s, S)\)-policy for our model is shown in Section 7.4. In Section 7.5 we use the analytic results to evaluate the impact of different return flow characteristics numerically. Section 7.6 concludes the chapter by discussing possible extensions to the model and delineating the scope of the results.
7.1 A Basic Inventory Model with Item Returns

Following the above motivation we consider a standard single item stochastic inventory model extended with a stochastic inbound item flow. For the time being, item returns are assumed to immediately raise the serviceable stock level. For an illustration one may think of situations where returned products can be reused directly without major re-processing, such as reusable packaging (compare Table 3.1). However, we remark that this assumption is for notational convenience mainly and does not limit the generality of the model essentially. In Section 7.6 we show how a recovery process involving a positive leadtime can be incorporated in this setting following a standard state-redefinition approach.

For ease of presentation we consider a discrete time setting (see Section 7.6 again for extensions to a continuous time model). We assume the following sequence of events. At the beginning of each period the inventory level is reviewed. A decision is taken on procurement orders, which are delivered after a fixed leadtime of \( T \) periods. Subsequently, demand and returns arrive throughout the period. Any unsatisfied demand is backordered. Let

\[
D_n^+ = \text{demand in period } n; \\
D_n^- = \text{returns in period } n; \\
D_n = D_n^+ - D_n^-, \text{ net demand in period } n.
\]

We assume \((D_n)_{n \in \mathbb{N}}\) to be independent identically distributed as an integer random variable \( D \) and let \( p_i = P\{D = i\}, i \in \mathbb{Z} \). Note that this assumption allows for stochastic dependence between demand and returns within the same period. For example, replacement demand triggered by returns can be taken into account. In contrast, there is no dependence across periods, provided that both demand and returns are themselves i.i.d. sequences. We recall from the previous chapter that the relation between the demand and return process is one of the distinguishing elements between the different models in literature. Independence or a (possibly stochastic) time lag are the two major options that have been considered. We defer a more detailed discussion concerning the appropriateness of these assumptions to Section 7.6. To describe the system's state let

\[
Y_n = \text{net stock at the beginning of period } n \text{ before ordering; } \\
A_n = \text{replenishment order size in period } n; \\
X_n = \text{net stock at the beginning of period } n \text{ after ordering} \\
\text{and receipt of replenishments; } \\
I_n = \text{inventory position at the beginning of period } n \\
\text{defined as } Y_n + \text{outstanding replenishments.}
\]

As explained above, we do not include a disposal option and therefore restrict \( A_n \) to be nonnegative for all \( n \). Obviously, this assumption only makes sense if the average demand outweighs the average returns since otherwise the inventory level would increase to infinity.