Chapter 10

STOCHASTIC AND OTHER EXTENSIONS

In this concluding chapter, we extend the $DLP$ model/algorithm and associated decision support system in order to incorporate uncertainty of data. Motivation for $DLP$ optimization under certainty (stochastic extension of the model) can be found in Chapter 6, Subsection 6.2.2.

In Sections 10.1 and 10.2, we focus on the case of uncertainty in production and objective coefficients. The resulting model/algorithm—a new method for chance-constrained stochastic linear programming to which we attach the acronym $SLP$—is a close analog of the $DLP$ approach. It involves a synergistic combination of techniques of stochastic dynamic programming and techniques of linear programming.

$SLP$ can be implemented by building very directly on the $DLP$ decision support system of the previous three chapters and provides a case study in the extension of the techniques described there. The extension of the range planning model of Chapter 1 and its solution are described by way of illustration. In Section 10.3, a second comprehensive illustration for a highway maintenance model where data uncertainty is inherent to the problem is described and solved.

Further extensions are briefly outlined in Section 10.4. In Section 10.4.1, we discuss $SLP$ with recourse. This additionally incorporates uncertainty in right-hand side elements, for example, supplies and demands. Its combination of chance constraints and simple or complete recourse constitutes a fundamental new approach to stochastic linear programming that is likely to be the correct stochastic model to employ in many multiperiod, resource planning situations. $DLP$ with recourse is a special case, involving uncertainty in only the right-hand side elements.

The need to restrict some variables to $(0,1)$ or integer values was motivated near the conclusion of Section 6.3 and in Section 6.4. Stochastic data
and discrete variables also arise simultaneously in Section 6.4. We briefly touch on potential $DLP$ and $SLP$ extensions to address problems of this type. This is an open area of research and much challenging work remains to be done.

The complete $DLP$-and-Extensions set of possibilities is summarized in Tables 10.1 and 10.2 at the end of the chapter.

10.1 $SLP$ Model for Optimization under Uncertainty

The discussion is first motivated using a variant of the rangeland problem of Chapter 1. Then a basic form of the $SLP$ model is developed in Section 10.1.2, followed by its general statement in Section 10.1.3.

10.1.1 Motivation

In the range decision problem of Chapter 1, Section 1.1, suppose that the outcomes of actions are uncertain. For example, consider an acre in state $S_3 = (1,1,1)$ under action $SP$ (see Table 1.1). Whereas previously we assumed that the resulting state was $S_2 = (1,1,0)$ with certainty, let us now assume that with probability 0.8 the action is successful in reducing the third parameter from 'positive trend' to 'no trend', and with probability 0.2 the action is not successful and the ensuing state remains $S_3$. Similarly, when action $BSPSR$ is applied to state $S_1 = (0,0,0)$, we previously assumed a deterministic outcome, resulting in state $S_4 = (2,2,0)$. Now let us suppose that the outcome of the action is uncertain and depends on weather conditions or reliability of the workers implementing its treatments, and results in states $S_2$, $S_3$ or $S_4$, say with probabilities 0.2, 0.1 and 0.7, respectively. Similarly for other actions: $SBSPS$ applied to state $S_1$ attains state $S_2$ with probability 0.4 and $S_3$ with probability 0.6; $SBSS$ applied to state $S_2$ attains state $S_4$ with probability 0.8 and $S_2$ and $S_3$, each with probability 0.1.

When an action, for example, $BSPSR$, is applied to a state for which it is feasible, its cost is assumed to be given, as before, by Table 1.1. Previously the benefit from an action was determined by the known destination state. Now the benefit realized is uncertain and will depend on the benefit associated with the destination state together with the probability of attaining it. For example, when action $BSPSR$ is applied to a state for which it is feasible, namely, $S_1$, then it attains state $S_2$ with probability 0.2. The associated benefit is now 5.5 (the benefit previously associated with any action that achieves $S_2$) times the probability 0.2. Thus, the expected benefit when $BSPSR$ is applied to an acre of the rangeland resource (in state $S_1$) is

$$(0.2 \times 5.5 + 0.1 \times 5.0 + 0.7 \times 10.0) = 8.6$$

and the associated cost of the action is 12.0.