2. Newton’s Five Laws

As mentioned, we will here assume some knowledge of elementary mechanics and the concepts within that theory. As the reader progresses through the book, he or she will find these concepts treated in substantial detail.

2.1 Newton’s Laws of Motion

Newton introduced the basic laws of mechanics in an axiomatic form with associated lemmas and definitions. We reproduce the laws here in forms that closely resemble Newton’s original version.

Newton’s 1st law (the law of inertia):

A body remains in its state of rest or in uniform linear motion as long as no external forces act to change that state.

This law puts the state of rest and the uniform linear motion on an equal footing. No external force is necessary to maintain uniform motion. The motion continues unchanged due to a property of matter we call inertia.

Newton made this law precise by introducing definitions of the concepts momentum and mass. Momentum is the product of the velocity of a body and the amount of matter the body contains. That amount of matter is called the mass of the body and is a measure of its inertia. Instead of “mass” we shall occasionally use the more precise term “inertial mass”. The inertial mass then, is a measure of that property of an object which makes the object resist changes in its state of motion.

We can write

\[ p \equiv mv, \quad (2.1) \]

where \( m \) is the inertial mass of the particle, \( v \) its velocity, and \( p \) the momentum. By “\( \equiv \)” we mean “equal to, by definition”. Mathematically, we can express the law of inertia as

\[ \text{no external force } \Rightarrow \quad p = \text{constant vector.} \quad (2.2) \]
In this way the law of inertia coincides with the law of conservation of momentum for a particle.

It is by no means simple to grasp the physical content of the law of inertia. We have (almost by definition) no experience with objects that are not subject to some external influence. We can imagine placing an object, not acted upon by any force, in an otherwise empty astronomical space. Can we later by any sort of observation or experiment decide if that object is at rest, or in a state of uniform motion? The answer to – or rather an analysis of – this question is a central theme in this book.

Newton’s 2nd law:

The change in the momentum of a body is proportional to the force that acts on the body and takes place in the direction of that external force.

By change one means here the time derivative of the momentum. The law can be expressed as

\[ \frac{dp}{dt} = F, \]  \hspace{1cm} (2.3)

where \( F \) is the impressed force. If we assume that the mass of the body is constant, Newton’s second law takes the form

\[ ma = F, \]  \hspace{1cm} (2.4)

where \( a = \frac{dv}{dt} = \dot{v} \) is the acceleration of the body.

In a given (orthogonal) cartesian coordinate system, Newton’s second law can be written in component form (see the Appendix):

\[ m \frac{d^2x}{dt^2} = F_x, \]
\[ m \frac{d^2y}{dt^2} = F_y, \]
\[ m \frac{d^2z}{dt^2} = F_z. \]

In the form (2.4) the law states that an impressed force \( F \) causes an acceleration \( a \) which is directly proportional to that force; the constant of proportionality \( |F| / |a| \) is the inertial mass \( m \) of the object.

The whole concept of force has been the subject of much debate since it was introduced by Newton. Let us here note the following: the second law [(2.3) or (2.4)] should not be considered as the definition of the concept of force. An essential feature of the law is that the force acting on the particle is supplied by a force law separate from (2.3) or (2.4). One example of such a force law is the law of gravity. Other forms are shown throughout the examples in this chapter.