4. The Galilei Transformation

Our own galaxy is called the Milky Way. The galaxy closest to us is the Andromeda Galaxy. Light from the Andromeda reaching us today initiated out there some 2,000,000 years ago. About this time man was at the beginning of his long journey through evolution.

Let us briefly restate Newton's first three laws of motion:

1. Law of inertia.
2. $mdv/dt = F$.
3. Law of action and reaction.

As soon as these laws are written we are faced with the following question: *relative to which coordinate system should we measure positions, velocities, and accelerations?* Without an answer to this question the laws lose their meaning.

Should we measure relative to a coordinate system fixed on the surface of the Earth? We know that the Earth revolves once around its axis every 24 hours and orbits the Sun in an approximately circular orbit every year.
The tangential velocity of the Earth in its orbit around the Sun is $v = 3 \times 10^4 \text{ m s}^{-1} = 30 \text{ km s}^{-1}$. The radius in that orbit is $r = 1.5 \times 10^{11} \text{ m}$. Thus, the Earth is accelerated towards the center of the Sun with an acceleration of magnitude $a = \frac{v^2}{r} = 6 \times 10^{-3} \text{ m s}^{-2}$.

A coordinate system at rest relative to the surface of the Earth (say a laboratory) clearly is in a complicated state of motion relative to the Sun. Should we instead measure the kinematical quantities relative to the Sun? The center of the Sun moves in an approximately circular orbit around the center of our galaxy with a velocity of 250 km s$^{-1}$. The distance of the Sun from the center of the galaxy is about 30000 light years ($\approx 2.8 \times 10^{20} \text{ m}$). The acceleration of the Sun with respect to the center of the galaxy is then $a = \frac{v^2}{r} = 2.2 \times 10^{-10} \text{ m s}^{-2}$.

Is it the center of our galaxy or perhaps the center of the Andromeda galaxy which should be taken as the center of our reference frame? Attempts to answer the questions of which coordinate system should be the basis for measurements of positions, velocities, and accelerations will throw light on the basic strengths and weaknesses of the Newtonian world picture, and enable us to understand the profound revision of the problem of motion which was initiated with Einstein's special and general theory of relativity.

Newton answered the question himself by adding to his three basic laws the two postulates: the postulate of absolute time and the postulate of absolute space (see Chapter 2). These two postulates therefore are of crucial importance to the understanding of Newton's mechanics.

We shall, however, later see that Newtonian mechanics — in spite of its enormous success — contains an intrinsic weakness. To perceive this, we shall for a while forget the postulate about absolute space and through the experience we gain in applying the equation of motion, try to find which coordinate systems may be used for a description of the phenomena of motion in Newtonian mechanics.

We begin our discussion with one of the basic tools for comparing measurements taken by observers in uniform motion with respect to each other.

### 4.1 The Galilei Transformation

By an *inertial frame* we understand a reference frame within which Newton’s three laws of motion, in their simple classical form, are valid.

In a certain sense the law of inertia is contained within the second law. When there is no force on a body its acceleration (relative to absolute space!) is zero, from which it follows that the velocity is constant. Newton used the law of inertia to explicitly state that no force is required to maintain uniform motion.