Some Aspects of Spin Gap in One- and Two-Dimensional Systems

N. Nagaosa

Department of Applied Physics, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113, Japan

Abstract We study some aspects of the spin gap state in one- and two-dimensional systems. We first give a description of the spin gap state in the double-chain system in terms of the Abelian bosonization. Next we interpret this spin-gap state in terms of the fermion language, and possible interpolation between one- and two-dimensional systems is discussed. Lastly we study the effect of the nonmagnetic impurities to clarify the similarity and difference between one- and two-dimensional systems with spin gap.

1. Introduction

Recently the formation of the spin gap has been the subject of intensive studies because it is observed commonly in high-$T_c$ cuprates, heavy fermions (Kondo Insulator), and one-dimensional magnets (double-chain systems and Haldane system). An interesting idea proposed by Rice et al. [1] is the interpolation from one- to two-dimensions by increasing the width of the system which can be realized in a particular class of materials. In this respect it is important to study first the one-dimensional spin gap state and next its possible generalization to two-dimensional systems. In this paper I present some studies along this line. The plan of this paper is the following. In section 2 the standard Abelian bosonization method is applied to the double-chain system and the fixed point corresponding to the spin gap state is discussed for both half-filled and doped cases. In section 3 a simple interpretation of the spin-gap state at half-filling is given in terms of spinon (fermion), and its relation to the $\pi$-flux state or the d-wave pairing state in 2D is discussed. In section 4 the comparison of the physical properties, i.e., magnetic susceptibility and the residual resistivity, are made between the one- and two-dimensional systems, and the similarity and difference between them is pointed out.

2. Abelian Bosonization Study of Double-Chain Systems

The double-chain systems have been recently studied extensively by several authors [2-7], but the physical picture for the spin gap state still remains obscure. In this section we give a simple picture for this state. We start with the following Hamiltonian in the Abelian bosonization formalism.

$$H = H_A + H_B + H_{t\perp} + H_{J\perp},$$

where

$$H_i = v_p \int dx \left[ \frac{1}{4\pi\eta_p} \left( \frac{d\phi_+^i}{dz} \right)^2 + \pi\eta_p P_+^{(i)\perp} \right] + v_p \int dx \left[ \frac{1}{4\pi\eta_p} \left( \frac{d\phi_-^i}{dz} \right)^2 + \pi\eta_p M_+^{(i)\perp} \right],$$

with $i = A, B$ being the chain index and...
\[ H_{t\perp} = -t_{\perp} \int dx \left[ \psi_{Re}^{(A)}(x) \psi_{Re}^{(B)}(x) + \psi_{Le}^{(A)}(x) \psi_{Le}^{(B)}(x) + h.c. \right], \quad (3) \]

\[ H_{J_{\perp}} = -J_{\perp} \int dx \vec{S}_A(x) \cdot \vec{S}_B(x), \quad (4) \]

\( \theta^{(i)}_+(\theta^{(i)}_-) \) is the phase variable describing the charge (spin) degrees of freedom, and \( P^{(i)}_+(M^{(i)}_+) \) is its canonical conjugate momentum. \( P^{(i)}_+(M^{(i)}_+) \) is related to the phase variable \( \theta^{(i)}_-(\theta^{(i)}_+) \) as \( P^{(i)}_+ = -\frac{d\theta^{(i)}_+}{dx} / 2\pi \) (\( M^{(i)}_+ = -\frac{d\theta^{(i)}_-}{dx} / 2\pi \)). In terms of these phase variables, the field operators of the electrons are represented as

\[ \psi_{Re}^{(i)}(x) = \frac{1}{\sqrt{2\pi \alpha}} \exp \left[ ik_F x + \frac{1}{2} (\theta^{(i)}_+ + \delta^{(i)}_+ + \sigma(\phi^{(i)}_+ + \phi^{(i)}_-)) \right], \quad (5a) \]

\[ \psi_{Le}^{(i)}(x) = \frac{1}{\sqrt{2\pi \alpha}} \exp \left[ -ik_F x - \frac{1}{2} (\theta^{(i)}_- - \theta^{(i)}_+ + \sigma(\phi^{(i)}_- - \phi^{(i)}_+)) \right]. \quad (5b) \]

Using these expressions the interchain interactions \( H_{J_{\perp}} \) is written in terms of \( \theta^{(i)}_\pm \) and \( \phi^{(i)}_\pm \) as

\[ H_{J_{\perp}} = -J_{\perp} \int \frac{\pi}{4} \sin \phi^{(A)}_+ \sin \phi^{(B)}_+ \cos(\theta^{(A)}_+ - \theta^{(B)}_+), \quad (6) \]

where the rapidly oscillating parts with the wavenumber \( \pm 4k_F \) does not contribute to the integral. Let us first consider the pure spin model at half-filling. The spin gap states are described as the massive phase of \( \phi^{(A)}_+ + \phi^{(B)}_+ \) and \( \phi^{(A)}_- + \phi^{(B)}_- \) for both F and AF \( J_{\perp} \) [2]. This is because \( s^{(A)}_+ s^{(B)}_- + s^{(A)}_- s^{(B)}_+ \) gives rise to \( \cos(\phi^{(A)}_- - \phi^{(B)}_-) \) and \( s^{(A)}_+ s^{(B)}_+ \) gives rise to \( \cos(\phi^{(A)}_+ + \phi^{(B)}_+) \) and \( \cos(\phi^{(A)}_- - \phi^{(B)}_+) \). It is impossible to fix both the canonical conjugate pair \( \phi^{(A)}_+ - \phi^{(B)}_- \) and \( \phi^{(A)}_- - \phi^{(B)}_+ \), and the spin gap state is realized by fixing \( \phi^{(A)}_+ - \phi^{(B)}_- \) and \( \phi^{(A)}_- + \phi^{(B)}_+ \). It is easily seen that there appears no spin moment (\( \langle S^{(A)}_z \rangle = \langle S^{(B)}_z \rangle = 0 \)) in this state.

Now let us turn to the doped case. We are interested in the situation where \( J_{\perp} \) is reasonably large, and the electron number is near the half-filling, i.e., near the Mott insulator. Let \( \delta \) be the concentration measured from half-filling. Then the characteristic energy due to the phase fluctuation \( \theta^{(i)}_\pm \) is \( \max(t, t_{\perp}) \delta \) where \( t \) is the intrachain hopping. On the other hand, the stabilization energy due to the spin gap formation is of the order of \( |J_{\perp}| \). Hence if \( \max(t, t_{\perp}) \delta \ll |J_{\perp}| \), it is allowed to consider first the exchange interaction \( H_{J_{\perp}} \) and later treat the interchain hopping \( H_{t_{\perp}} \) as a perturbation.

In the case of \( t_{\perp} = 0 \), the effect of the doping is summarized in the factor \( \cos(\theta^{(A)}_+ - \theta^{(B)}_+) \) which appears in eq. (6). If this factor gives the finite expectation value, i.e., the combination \( \theta^{(A)}_+ - \theta^{(B)}_+ \) is fixed and massive, the dynamics of the spin phases \( \phi \)'s remains essentially the same and the spin gap persists. On the other hand, if the spin gap