Fig. 19. Bifurcations of stationary regimes (A.A. Andronov et al. (1959), Fig. 175, Chapter IV, § 5)

Fig. 20. Relaxation oscillations (A.A. Andronov et al. (1959), Fig. 176, Chapter IV, § 5)

Fig. 21. A bifurcation diagram (from The Theory of Indirect Regulation by Andronov, Bautin and Gorelik (1946))

ogy of the catastrophe theorists) in relation to a generic curve with a point of inflection.

Numerous bifurcation diagrams of two-parameter and three-parameter families can be found, for example, in an article by A.A. Andronov, N.N. Bautin and G.A. Gorelik “A theory of indirect regulation” (1946). The cusp here (Fig. 14 of the article is reproduced as our Fig. 21) is related to a special, and not fully smooth, system, but the authors note that the structure of the parameter space is “in a qualitative way not specific” for the case they consider.

§ 5. Physicists’ Treatment of Catastrophes Before Catastrophe Theory

5.1. Thermodynamics. Physicists have always made use of constructions more or less equivalent to catastrophe theory in investigations of concrete problems. In this sense, catastrophe theory may be compared with mathematical analysis.
Without the help of analysis, Huygens was able to solve the majority of problems solved by Newton. But such solutions required the genius of Huygens while, nowadays, the same problems may be solved with the help of analysis by any student. In exactly the same way, mastery of the technique of singularity theory allows one to obtain results automatically that otherwise require inventiveness and substantial efforts of imagination, simultaneously extending them to more complicated situations where “elementary” methods would lead to vast calculations. Porteous (1974) in an article “Nobel prizes for catastrophes” points out many examples of acknowledged physical achievements, whose authors, independently of one another, used ideas formalized later in singularity theory.

These ideas were systematically used in thermodynamics from the time of J.C. Maxwell and, especially, J.W. Gibbs. The perestroika (Fig. 22) of the isotherms of van der Waals’ equations of state is a typical example of an application of the geometry of the pleat. An analysis of the asymptotics in a neighborhood of the critical point quickly leads to the understanding that this geometry is independent of the exact form of the equations of state. This fact was well known in the time of Maxwell, and is mentioned in most thermodynamics textbooks (for example, in the book of L.D. Landau and E.M. Lifshits (1964, § 84)).

Maxwell’s idea was to draw the horizontal portion of an isotherm so that the areas of its lunes lying above and below it coincide. This equality implies that the transition from one of two competing minima of the potential to the other occurs at that moment when the second one becomes lower. Therefore, the equality just pointed out is called Maxwell’s convention in catastrophe theory, too. A specific bifurcation diagram (formed of those values of the parameters at which the potential has two minima of equal height) corresponds to it. Therefore, even in the more general situation of the theory of singularities (and also for complex-valued functions), the set of values of the parameters at which a function of a family has several critical points with a common critical value is called the Maxwell stratum. For example, for a two-parameter family the typical singularity of the critical value as a (multivalued) function of the parameters is the swallowtail, and the Maxwell stratum corresponds to the curve of self-