A SOFTWARE PACKAGE TOOL FOR MARKOVIAN
COMPUTING MODELS WITH MANY STATES:
PRINCIPLES AND ITS APPLICATIONS

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ABSTRACT
A computing system can be formulated by modeling a continuous-time Markov chain with many
states, and be evaluated by using the reliability/performance measures. The randomization tech­
nique is discussed to derive the transient solution for a Markov chain. A software package tool
is implemented by using the randomization technique and introducing a new idea of identifying
when the transient solution converges to the steady-state solution in advance. Numerical examples
are illustrated by using our software package tool to evaluate the optimal maintenance policies
for computing systems. Some interesting maintenance policies are suggested from the numerical
examples.

1. INTRODUCTION
It is of great interest and importance to operate a computing system with high reliability and
performance [1, 2]. To evaluate such a system, we should derive analytically and/or numerically
reliability/performance measures by formulating a stochastic model of the system [3, 4, 5]. A
continuous-time Markov chain is one of the most powerful stochastic processes to analyze the
system. In particular, we are very much interested in a continuous-time Markov chain with many
states since modeling a Markov chain yields many states in practice [6]. We develop a software
package tool for calculating the transient state probabilities for a continuous-time Markov chain
with many states. Several performance/reliability measures can be calculated by using the state
probabilities.

We adopt the randomization technique [7] for calculating the transient state probabilities as
well as the steady-state probabilities. It is assumed that the transient state probabilities converge
the steady-state probabilities as time tends to infinity under certain assumputions. In principle, it
is possible to calculate the transient and steady state transition probabilities. However, it is quite
difficult to do so if there are many states such as some hundreds or thousands of states.

For our software package tool, we specify the initial state probability vector \( \pi(0) \). Once the
initial state probability vector \( \pi(0) \) is specified, we can calculate the transient state probability
vector \( \pi(t) \) at time \( t \). However, it is quite difficult in advance to identify when the transient
state probability vector converges to the steady-state probability vector with enough precision. We propose a new idea of calculating the convergence time \( t_s \) of the steady-state probability in advance from the knowledge of the randomization technique.

In this paper, we discuss our software package tool and its applications. In §2 we discuss the randomization technique for calculating the transient solutions as well as the steady-state solutions for a continuous-time Markov chain with many states. We propose a new idea of the convergence time which will be implemented in our software package tool. We further present two examples of maintenance policies for a computing system in §3, and show how our software package tool is useful. The first example is maintenance policies based on retries for a computing system. Calculating the transient and steady-state availabilities, we can obtain the effective maintenance policies. The second example discusses maintenance policies for a hardware and software system. We propose two software maintenance policies for a two-unit hardware system and compare them.

2. MATHEMATICAL PRELIMINARIES

2.1 Randomization Technique

Let us briefly sketch the randomization technique (see Ross [6], pp 141-183). There are several techniques of calculating the exponential of matrix [8], since they are quite famous as the eigenvalue problems of the matrices.

Consider a continuous-time Markov chain with \( N \) states. Let

\[ \pi(t) = \{\pi_1(t), \pi_2(t), \ldots, \pi_N(t)\} \]

be the state probability vector at time \( t \), where the initial state vector

\[ \pi(0) = \{\pi_1(0), \pi_2(0), \ldots, \pi_N(0)\} \]

is prespecified. Let \( Q \) be the infinitesimal generator for the continuous-time Markov chain. Then, the matrix differential equation is given by

\[ \frac{d\pi(t)}{dt} = \pi(t)Q, \]

where the initial condition \( \pi(0) \) is given. Note that each element of the infinitesimal generator is given by

\[ q_{ij} = \lim_{\Delta t \to 0} \frac{P\{X(t+\Delta t) = j|X(t) = i\}}{\Delta t}, \quad (i \neq j) \]

\[ q_{ii} = -\sum_{i \neq j}^N q_{ij}. \quad (i = j) \]

It is easy to solve the matrix differential equation (3). We have

\[ \pi(t) = \pi(0)e^{Qt} = \pi(0)[I + \sum_{n=1}^{\infty} \frac{(Qt)^n}{n!}], \]

where \( I \) is an identity matrix.