3. Temporal Logic and Model Checking

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1. Reactive Systems and Temporal Properties

1.1 Example: The alternating bit protocol

Channels may drop (or perhaps duplicate).
- Sender retransmits (at some interval) until matching ack received.
- Sequence numbers prevent duplication of msgs or acks.
- Sequence numbers are modulo 2 (hence, “alternating bit”).

This is an example of a “reactive” system [1] (Pnueli):
- “Reacts” to stimulus from environment.
- Does not terminate.

Note each component (sender, receiver, channels) is also a reactive system.

1.2 Temporal properties

To reason about reactive systems and the interaction of their components, we need to be able to state temporal properties.

E.g., for the alternating bit protocol:
- Every message sent is eventually received.
- A message is not received unless one is sent
- If x is sent before y, then x is received before y.

Some properties of the components:
- Sender continues to resend msg until ack.
- If channel continues to receive input, it eventually transmits (does not drop) a msg.
- Recvr does not produce ack before msg is output.
- etc.

Note: these are properties about relationships in time (i.e., temporal properties).
1.3 Formalizing temporal properties

... to specify and reason about reactive systems.

- Consider using first order logic to write temporal properties, representing time by a natural number $t$.
  For example: “every time an $x$ msg is input, one is eventually output”

$$\forall t \geq 0 : \text{input}(x, t) \Rightarrow \exists t' \geq t : \text{output}(x, t')$$

This is adequate, but a bit hard to read!

- Temporal Logic
  Pnueli suggested using temporal logic to express properties of reactive systems. In temporal logic, the time parameter $t$ is implicit:
  - $Gp$ true at time $t$ if $p$ is true at all $t' \geq t$.
    \[ \begin{array}{ccccccc}
    p & p & p & p & p & p & p \ldots \ni Gp \ldots \\
    \end{array} \]
  - $Fp$ true at time $t$ if $p$ is true at some $t' \geq t$.
    \[ \begin{array}{ccccccc}
    p & p & p & p \ldots \ni Fp \ldots \\
    \end{array} \]

Note, $G$ and $F$ are dual:

$Gp \equiv \neg F \neg p$

$Fp \equiv \neg G \neg p$

Here are, for example, some other equivalences:

$Gp \land Gq \equiv G(p \land q)$

$Fp \lor Fp \equiv F(p \lor q)$

But note,

$Gp \lor Gq \neq G(p \lor q)$

$Fp \land Fp \neq F(p \land q)$

Our previous example in temporal logic:

$G(\text{input}(x) \Rightarrow F \text{output}(x))$

This can be read “always, if input$(x)$ then eventually output$(x)$.” It is an example of a liveness property, since it states some “good” condition that must eventually occur.

- “Infinitely often” properties
  - Note $G Fp$ means that $p$ occurs infinitely often (“always eventually $p$”).
    This is equivalent by De Morgan’s laws to $\neg F G \neg p$ or “a point is never reached where $p$ is forever false”.
