5. The Axiom of Uniform Stress

5.1 The Undisturbed Design

A German forester (Metzger 1893) found that the tapering typical of spruce trees will ensure a uniform distribution of the surface bending stresses, if the stem diameter, D, is cubical relative to the distance, h, from the effective point of application of the wind load (Fig. 23).

The sudden change in stem diameters in the area of branch joints is a vivid example of the load-distributing, into-the-ground downstem fiber arrangement and force-flow orientation of the branches. Above and below the branch joints one finds stresses of equal intensity.

The axiom of uniform stress manifests itself conspicuously in the area of the roots. According to Mohr-Coulomb’s law, the shear strength of the ground increases in proportion to the degree of shear surface compression. This increase may be compared to the increase in friction proportionately to the growing pressure of the friction surfaces against each other. The wind load that acts on the tree is caught by the crown (sail) and passes the trunk (mast) to enter the ground. The ground is compressed on the lee side and lifted on the windward side (Fig. 24). Regardless of the fact that the shear strength on the lee side increases while that on the windward side is reduced, the weaker side of the ground must bear almost equally intense stresses because the weight acting on the ground is negligible compared to the wind force that acts higher up. The less shear-resistant ground on the windward side must be reinforced by additional, longer roots, which are not necessarily found on the stronger lee side. Since the roots on the windward side are exposed to higher loads, the bending stiffness of their cross sections is for the most part greater than that of the roots on the lee side.

The axiom of uniform stress can also explain the shapes of root cross sections as being the sums of all tree ring width distributions. Figure 25 shows an I-beam-shaped root as the result of pure bending loads. It grew strongest on the highly stressed compression and tension sides of bending and developed least in the middle, i.e., in the area of the neutral fiber of bending, where bending stresses are absent. Buttress roots preferably develop on the windward side (Mattheck 1991) in the presence of sinker roots that join the horizontal roots at a greater distance from the stem. They are the result of superimposed tensile stresses and bending stresses that add up on the upper side and are partly or completely neutralized on the lower sides of the roots. It is due to the resulting decreasing stress distribution that the upper tree rings
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\[ \sigma_0 = \frac{F_{\text{wind}} h}{\pi D^3} \approx \text{constant} \]

\[ h = \frac{\pi \sigma_0}{32 F_{\text{wind}} D^3} \]

**Fig. 23A,B.** Stem-tapering defined by the requirement of uniform bending stresses. A Tapering of spruce stems (Metzger 1893). B Local change in diameter due to lateral introduction of the wind load by a main branch into the stem.

**Fig. 24.** Root morphology defined by the uniform stress axiom and by Mohr-Coulomb’s law.