I. Methods in the Theory of Sheaves and Stein Spaces

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Introduction

This article is devoted to cohomological methods in complex analysis, which have undergone intensive development in the course of the last 35 years. The basic object of study here is a complex analytic space or, roughly speaking, a complex analytic manifold with singular points. Analytic spaces belong (as do also analytic and differentiable manifolds, supermanifolds, and algebraic varieties) to the class of mathematical structures which are defined by fixing on a given topological space a certain stock of continuous local functions. An adequate means for describing such a structure is the concept of a sheaf, which is discussed in Chapter 1. In Chapter 2 we consider first the technically convenient concept of a ringed space, i.e., a space endowed with a sheaf of rings (or algebras), and then a particular case of it—the concept of a complex analytic space, which is fundamental for what follows. We also define here a coherent analytic sheaf. In Chapter 3 we discuss the theory of cohomology with values in a sheaf of abelian groups and indicate its simplest applications to problems of analysis, for example, to the solution of the Cousin problems for poly-cylindrical domains. In Chapter 4 we give a survey of results related to the so-called Stein spaces. These remarkable complex spaces, which can be defined, roughly speaking, as spaces with a very large stock of global analytic functions, emerged historically as the first objects in complex analysis on which the methods of cohomology were tried.