Linkage of Viscous and Inviscid Boundary Element Methods

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Summary

Justifiable approximations of the vorticity distribution in certain high Reynolds number viscous flows are identified. The approximations are utilized to simplify integral representations of viscous flow equations and to develop boundary element procedures previously obtained through the inviscid flow idealization.

Introduction

For a number of years, the present author and his co-workers have been developing a numerical method for the solution of flow problems involving large scale viscous separation. With this method, the differential equations describing the motion of a viscous fluid are first recast into the form of integral representations, which are then solved numerically [1]. The integral representation in general is composed of two integrals, one over the fluid domain and the other over the boundary of the fluid domain. The integral representation method is thus a generalized version of the boundary element method that deals with boundary integrals only. Boundary element procedures in common use today, e.g., the panel procedures, are based on the inviscid fluid idealization. The integral representation method, in contrast, considers the flow of a viscous fluid. In the present paper, it is shown that the inviscid boundary element procedures are obtainable as approximations of the integral representation method.

Anatomy of High Reynolds Number Flow

Consider the flow of an incompressible viscous fluid past the exterior of a finite solid body such as an airfoil. In such a flow, the vorticity is continually generated on the surface of the solid and transported in the fluid by both diffusion and convection. There exists a region surrounding and trailing the solid where the vorticity is present and hence the viscous effects are important. Under general circumstances, the viscous
region in a high Reynolds number flow is composed of a boundary-layer zone, a recirculation zone, and a wake zone. The present paper, however, considers circumstances where no appreciable recirculation zone is present. For two-dimensional flows, the vorticity in the near wake is "shed" from the boundary layer zone near the trailing edge of the airfoil. This trailing edge wake tends to roll up, forming clusters of concentrated vorticity. In unsteady flows associated with lifting bodies undergoing sudden changes of motion, concentrated dosages of vorticity, such as the starting vortex and the 'Weis-Fogh' vortices [2], are shed near the trailing edge. In three-dimensional flows, tip vortices or leading-edge vortices also exist as a part of the near wake structure.

Boundary Integral Equations
Wu [1] has expressed the kinematics of the viscous flow problem as an integral representation for the velocity vector. This integral representation is composed of a Biot-Savart integral over the fluid domain and a boundary integral containing the velocity boundary conditions. For an external flow, the boundary integral yields simply $\vec{V}_\infty$, the freestream velocity. The integrand of the Biot-Savart integral contains vorticity. In evaluating the Biot-Savart integral, the vorticity content of the boundary layer is justifiably approximated by a vortex sheet enveloping the solid surface. This approximation is not an inviscid fluid idealization. It merely recognized the fact that, since the boundary layer is thin in a high Reynolds number flow, the distribution of vorticity across the thickness of the boundary layer is unimportant in the evaluation of the Biot-Savart integral. In the absence of an appreciable recirculation zone, the trailing edge wake and the Weis-Fogh vortices are justifiably approximated by a series of vortex filaments [2]. The integral representation for the velocity vector in a two-dimensional viscous flow, with the use of the approximation just described, yields the following equation at the solid boundary [2]:

$$
\oint_{S^+} \frac{\vec{\gamma}_0 \times (\vec{r}_o - \vec{r})}{|\vec{r}_o - \vec{r}|^2} \, dB_o = -\frac{1}{2\pi} \sum_k \frac{\vec{\gamma}_k \times (\vec{r}_k - \vec{r})}{|\vec{r}_k - \vec{r}|^2} + 2\pi \vec{V}_\infty \quad (1)
$$

where $\vec{r}$ is the location of a point on the solid surface $S$, $\vec{r}_o$ is the strength of the boundary-layer vortex sheet, $\vec{r}_k$ and $\vec{\gamma}_k$ are respectively the strength and location of the $k$th wake vortex and $S^+$ represents the location of the boundary layer. $S^+$ is at an infinitesimal distance from $S$. 

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