Extended Thermodynamics of Classical and Degenerate Ideal Gases

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For J. L. Ericksen on his 60th Birthday

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C. M. Dafermos et al., The Breadth and Depth of Continuum Mechanics
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1. Introduction

Extended thermodynamics is a field theory with the primary objective of determining the thirteen fields of mass, momentum and energy densities, stress deviator and heat flux. This distinguishes it from ordinary thermodynamics which knows only five basic fields, viz. the densities of mass, momentum and energy.

The thermodynamic theory of thirteen fields provides an improved description of rapidly changing processes with steep gradients in dilute gases. Constitutive equations are formulated for the fluxes of stress and heat flux, and the entropy principle as well as the principle of material objectivity are used to restrict the generality of the constitutive functions.

Such restrictions are so stringent that even the thermal equation of state, which relates pressure to density and temperature, cannot be arbitrary. There are only three known materials that conform to the requirements, viz. a classical monatomic gas and the degenerate gases of the BOSE and FERMI type.

The reader who wishes to appreciate the conceptual simplicity and the strongly restrictive results of extended thermodynamics is advised to read Sections 3, 4 and 5, to glance through Section 7 and to proceed from there to Section 8.3 where the essential results are listed in the equations (8.15) for classical ideal gases as well as for strongly degenerate BOSE and FERMI gases. The final form of the constitutive equations contains no more than two positive-valued functions of density and temperature and three constants of which two are additive and therefore unimportant.

The most amazing feature of the theory is that for a classical ideal gas it produces constitutive relations that are the same as those of GRAD's method of thirteen moments (see [1]) in the kinetic theory of gases, except that one—only one (!)—constant can still be adjusted.